

# Complete Controllable Distributed Testing

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# Challenges in Testing

- These include:
  - Scale
  - Concurrency
  - Distribution
  - The Oracle Problem (checking test output).
- Currently expensive, error-prone, mainly manual.
- Possible solution: model-based testing.

# Formal languages used

- Typically have states and transitions between states triggered by actions.
- Many based on one of:
  - Finite state machines (FSMs)
  - Labelled transition systems (LTSs)
- Tools might translate models to either FSMs or LTSs.

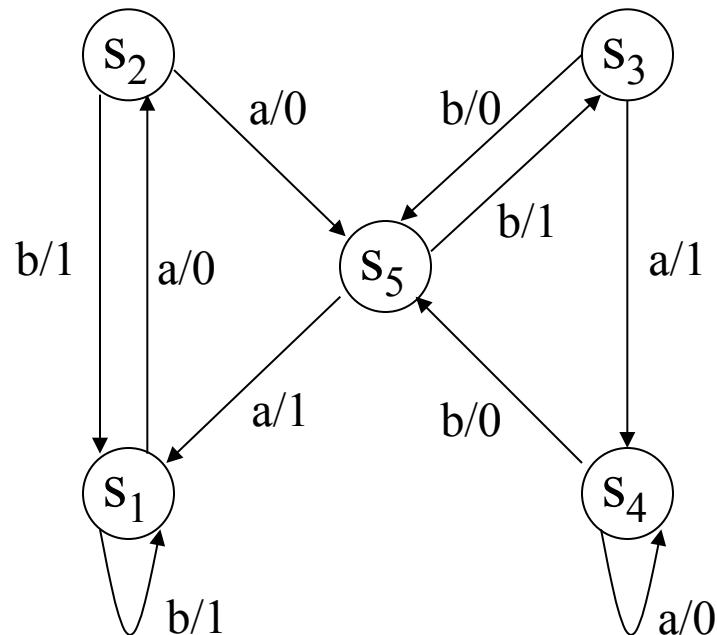
# Assumptions

- Usually we only observe interactions between the system under test (SUT) and its environment - *black-box testing*.
- To reason about test effectiveness we assume:
  - The behaviour of the SUT can be expressed in the same language as the model.
- This allows us to define *implementation relations* between models.

# Finite State Machines and MBT

# Finite State Machines

- The behaviour of  $M$  in state  $s_i$  is defined by the (regular) set of input/output sequences (traces) from  $s_i$



# Implementation relations

- Assuming all models are completely-specified, these are:
  - Equivalence for deterministic FSMs.
  - Language inclusion for nondeterministic FSMs.
- There are efficient algorithms for deciding these properties, so:
  - If we know that the SUT behaves like FSM  $N$  and we have specification FSM  $M$  then we can determine whether  $N$  conforms to  $M$ .
- We will focus on: *deterministic FSMs*.

# Fault Domains

- A set of models that represent potential behaviours of the system.
- Standard fault domains for testing from an FSM  $M$  with  $n$  states:
  - The SUT behaves like an unknown FSM  $N$  with at most  $n$  states.
  - The SUT behaves like an unknown FSM  $N$  with at most  $m$  states (some  $m > n$ ).



# Complete test suites

- A test suite  $T$  is  $m$ -complete when testing against  $M$  if:
  - For every FSM  $N$  with no more than  $m$  states, if  $N$  does not conform to  $M$  then there is a test sequence in  $T$  that demonstrates this.
    - Implicit: fixed input and output alphabets.
- If the SUT satisfies these conditions then such a test suite *determines correctness*:
  - If the SUT passes the test suite then either it is correct or has more than  $m$  states.

# Existence of $m$ -complete test suites

- We can produce an  $m$ -complete test suite:
  - For each FSM  $N$  with no more than  $m$  states we:
    - Determine whether  $N$  conforms to specification  $M$ .
    - If  $N$  does not conform to  $M$  then we add a test sequence that demonstrates this.
- These steps are computable (and there are finitely many FSMs to consider).

# Smaller test suites

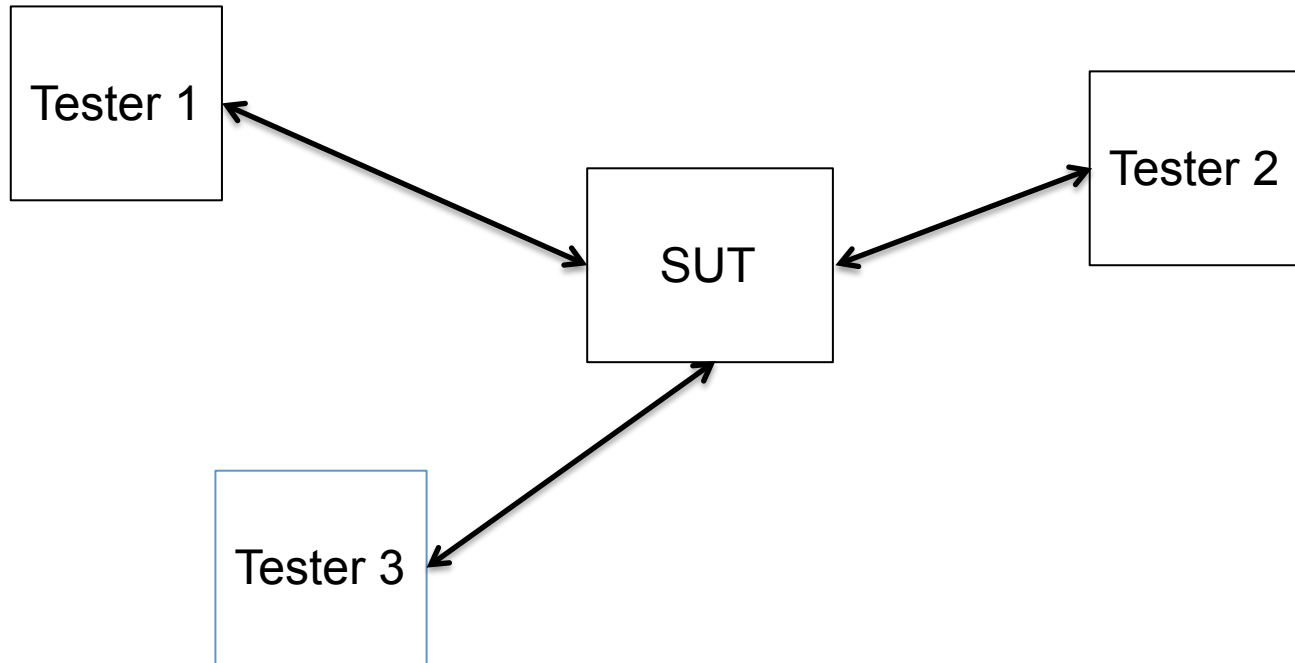
- There are more efficient algorithms.
- Many build test sequences from ‘parts’ that:
  - Reach a state  $s$ .
  - Distinguish two states  $s$  and  $s'$  (or distinguish every pair of states).
- For deterministic FSMs these ‘parts’ can be produced in low-order polynomial time.

# Summary: using a single tester

- For (deterministic) FSM specification  $M$ :
  - We can efficiently decide whether an observation is allowed by  $M$  (the Oracle Problem).
  - We can efficiently produce tests to reach states or distinguish states.
  - We can efficiently decide whether an FSM  $N$  conforms to  $M$ .
  - We can generate an  $m$ -complete test suite for  $M$ .

# Distributed Testing

# The Architecture

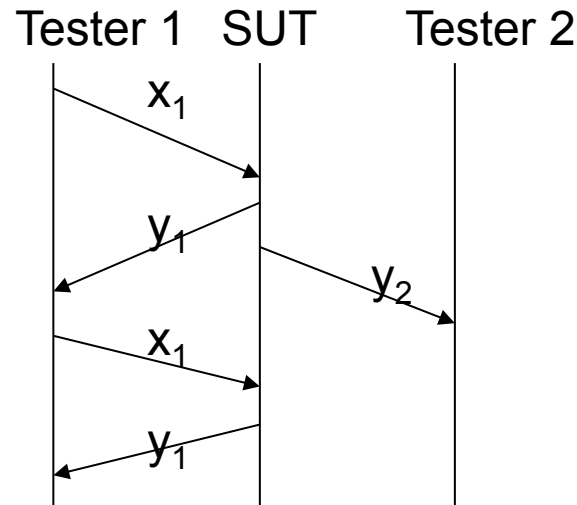


# Distributed testing

- We have:
  - An SUT that interacts with its environment at physically distributed interfaces (ports).
  - A tester at each port.
  
- Will focus on the case where:
  - The testers do not interact with one another during testing and there is no global clock.
  - The testers log their observations and logs are combined after testing.

# Consequences

- Each tester observes only the sequence of interactions (*local trace*) at its port

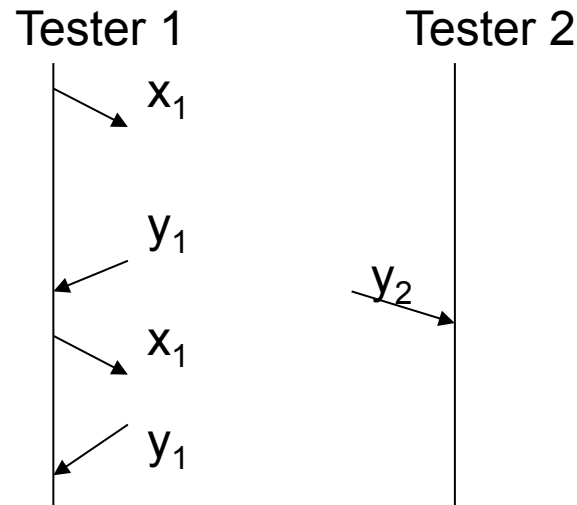


- The tester at port 1 observes  $x_1y_1x_1y_1$  and the tester at port 2 observes  $y_2$  only.



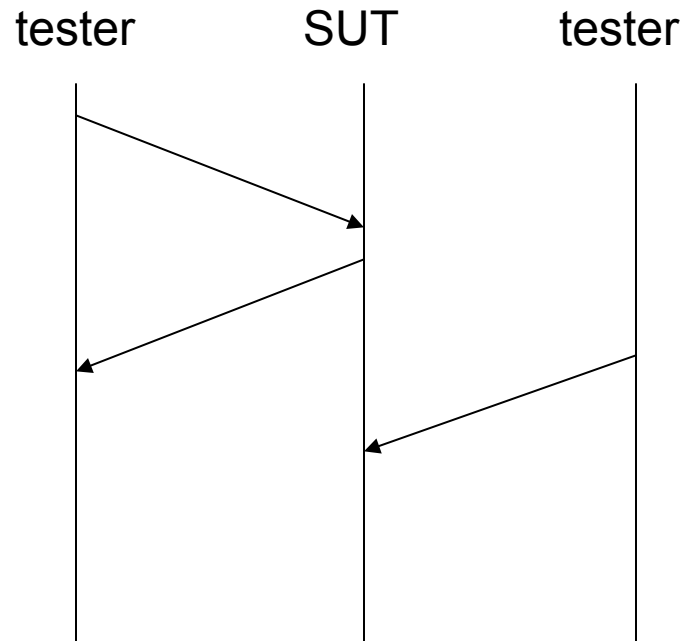
# What the testers observe

- Given global trace  $z$ , the tester at  $p$  observes a local trace  $\pi_p(z)$ .



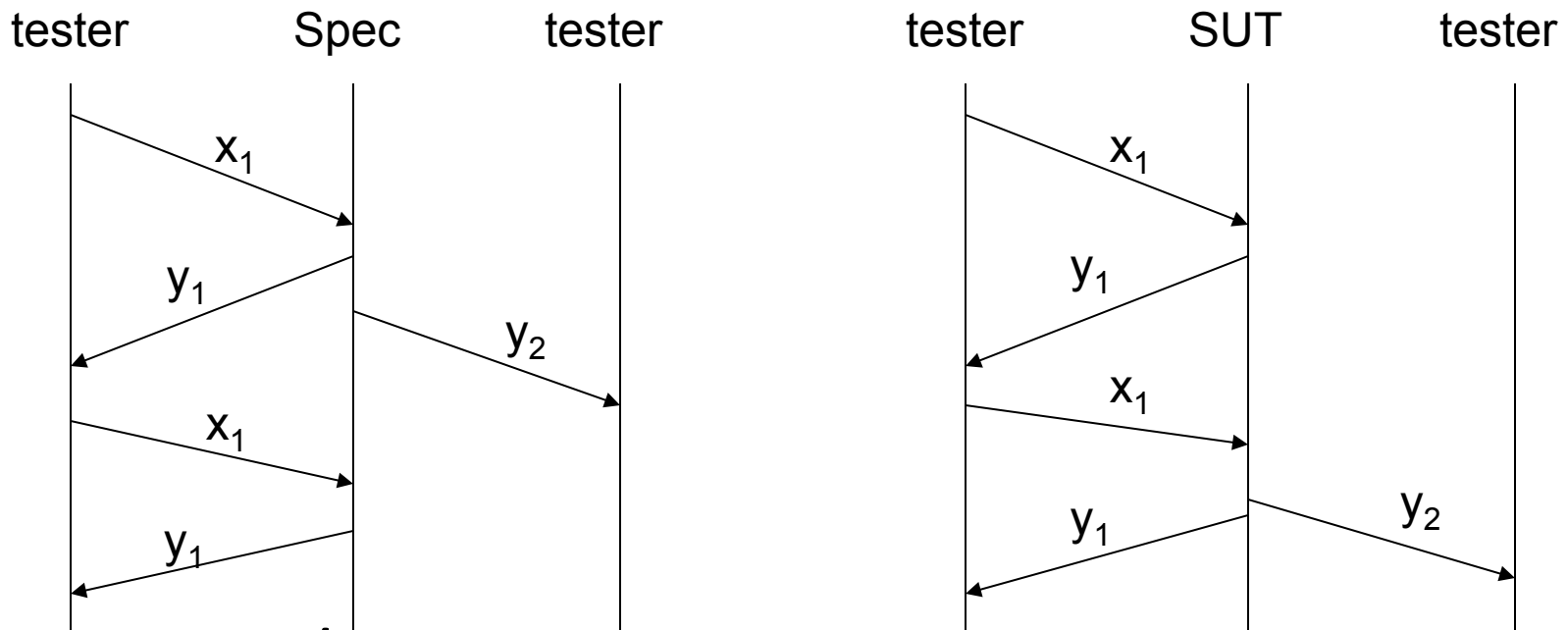
# Controllability problems

- This test has a controllability problem: introduces nondeterminism into testing.



# Observability problems

- The following look the same



- Testers/users cannot 'map' output to input

# Equivalent global traces

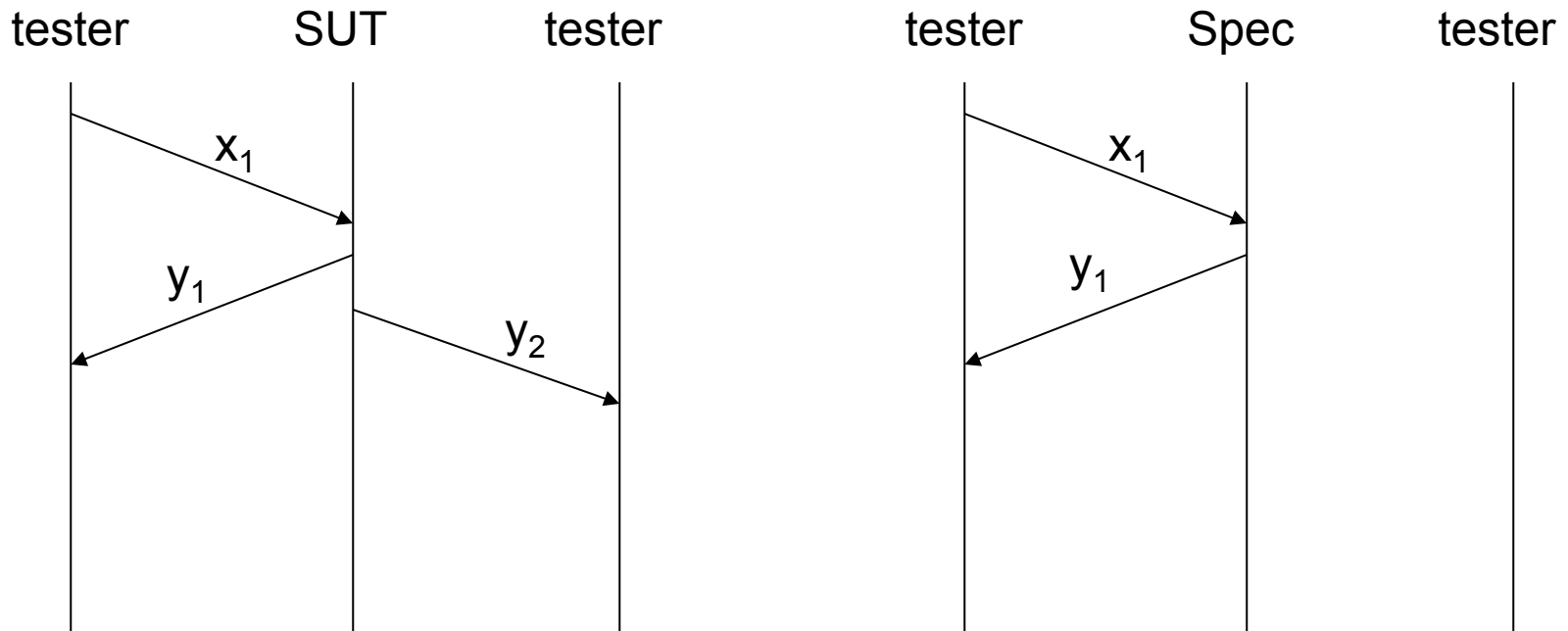
- Since we only observe local traces:
  - Global traces  $z$  and  $z'$  are indistinguishable if their projections are identical: the local traces are the same. We denote this:  $z \sim z'$

$$z \sim z' \Leftrightarrow \forall p \in \mathcal{P}. \pi_p(z) = \pi_p(z')$$

- The following are equivalent under  $\sim$ 
  - $x_1/(y_1, y_2)x_1/(y_1, -)$
  - $x_1/(y_1, -)x_1/(y_1, y_2)$
- Both have  $x_1y_1x_1y_1$  at port 1 and  $y_2$  at 2.

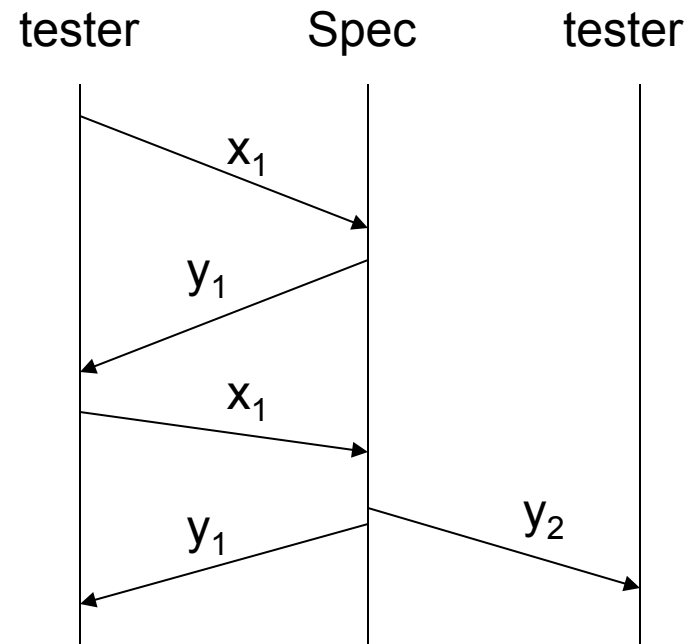
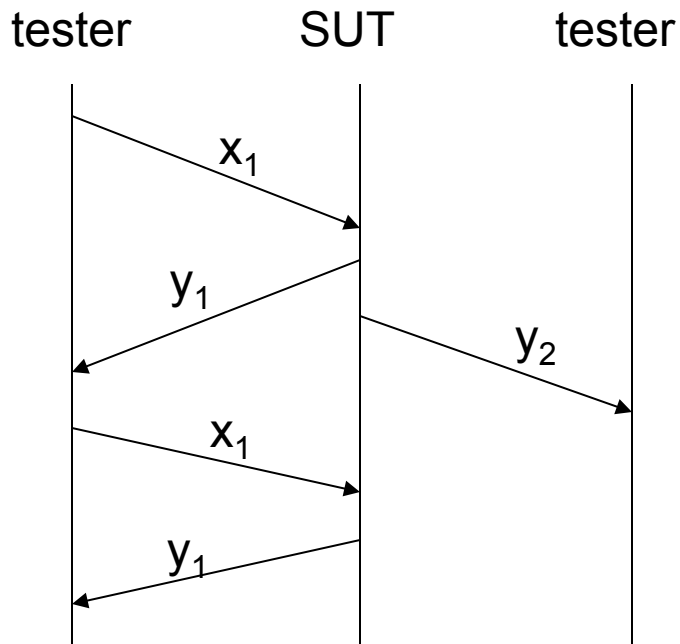
# A simple output fault

- Input  $x_1$  detects the fault.



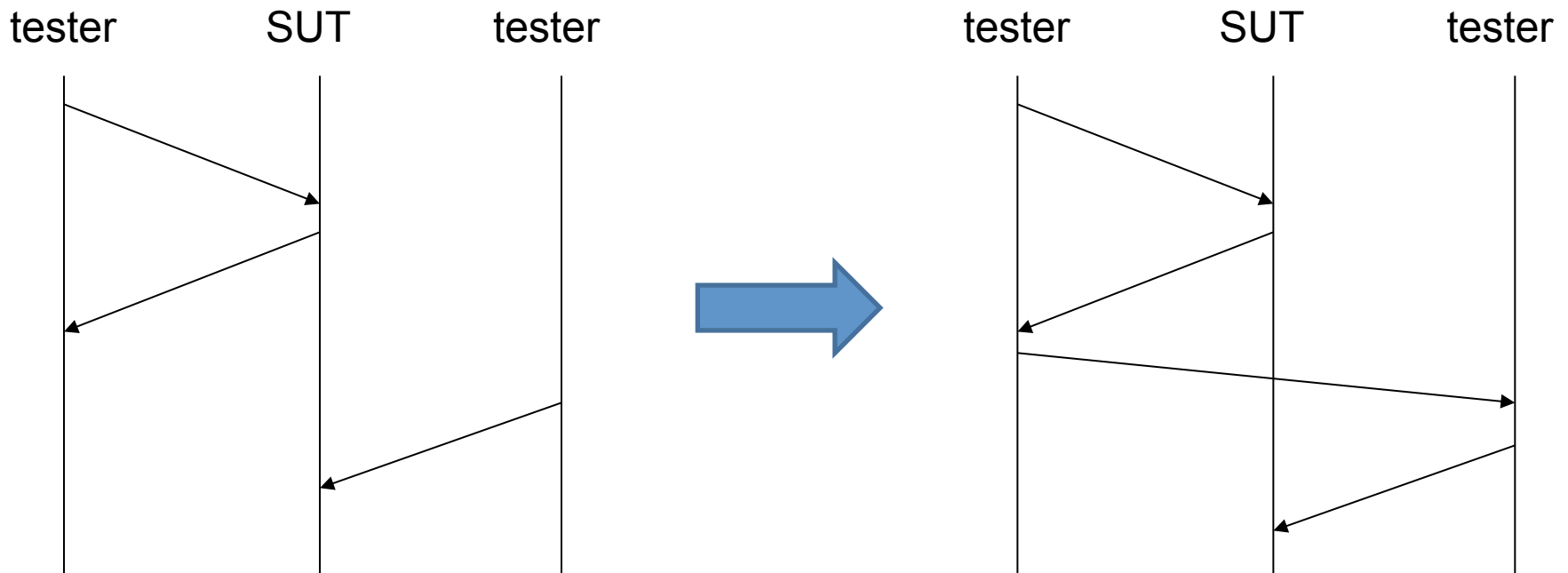
# Test effectiveness is not monotonic

- However:  $x_1x_1$  does not detect the fault.



# Using an external network

- *Sometimes* we can overcome controllability and observability problems.



# Distributed Testing and Deterministic Finite State Machines



# An allowed behaviour

- Given specification  $M$ , a trace  $\sigma$  is allowed if

$$\exists \sigma' \in L(M). \sigma' \sim \sigma$$

# An implementation relation for distributed systems

- We say that FSM  $N$  conforms to FSM  $M$  if:
  - Every global trace of  $N$  is indistinguishable from a global trace of  $M$ .

$$\forall \sigma \in L(N) \exists \sigma' \in L(M). \sigma' \sim \sigma$$

# The language defined by an FSM

- With distributed observations, this is:

$$\mathcal{L}(M) = \{\sigma' \mid \exists \sigma \in L(M). \sigma' \sim \sigma\}$$

- So, a behaviour is correct if

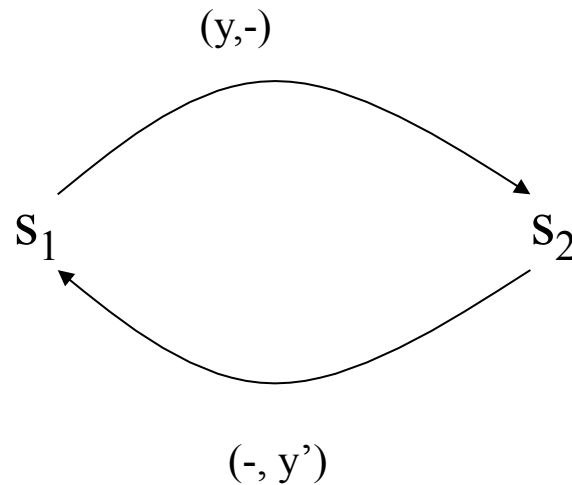
$$\sigma \in \mathcal{L}(M)$$

- N conforms to M if and only if

$$L(N) \subseteq \mathcal{L}(M)$$

# The language need not be regular

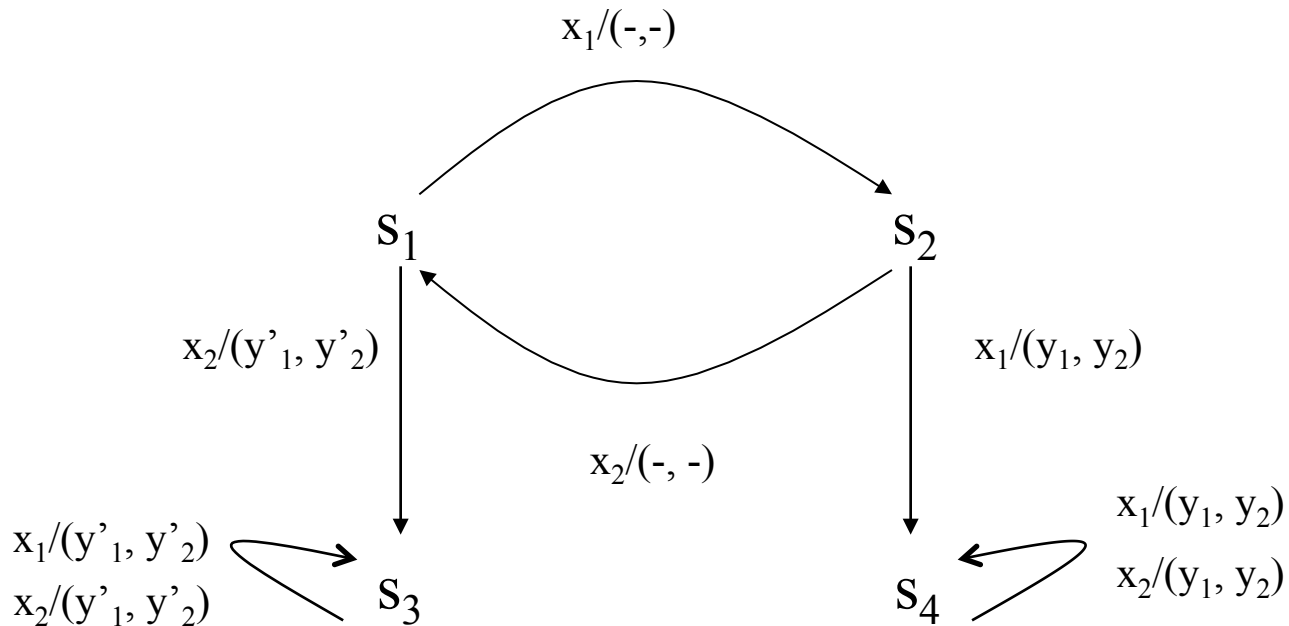
- The following ‘cheats’ – does not have any inputs.



- Clearly,  $\mathcal{L}(M)$  is not regular.

# The language need not be regular

- Following shows this (take the intersection with  $\{x_1^*\}\{x_2^*\}$ ).



# The Oracle Problem in Distributed Testing

- We observe projections

$$\sigma_1, \dots, \sigma_m$$

- We want to know whether the following holds:

$$\exists \sigma \in L(M). \forall p \in \mathcal{P}. \pi_p(\sigma) = \sigma_p$$

- Essentially, a membership problem.

$$\sigma_1 \dots \sigma_m \in \mathcal{L}(M)$$

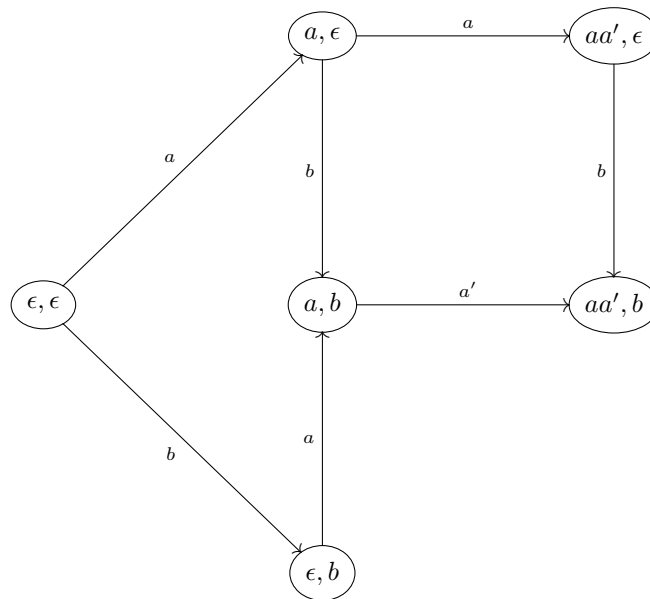
- It is *decidable*, since we could:
  - Form all interleavings of the projections.
  - For each such global trace, determine whether the global trace is allowed by the specification.
- This leads to a combinatorial explosion.

# Solving the Oracle Problem

- We observe projections  $\sigma_1, \dots, \sigma_m$
- We can form a finite automata whose language is the set of corresponding global traces (the oracle problem is then FA intersection).
- A state is a vector whose  $i$ th component is the latest event from  $\sigma_i$

# Example

- Two ports, local traces  $aa'$ ,  $b$ .





# How this works (1)

- We define a partial order  $<$  on events in  $\sigma$ :  $a < a'$  if (from the observations) we know that  $a$  must have been before  $a'$ .
- In this case:
  - Two events are related iff they are at the same port.
- Important property:
  - For  $a$  to occur we must have all events before  $a$  (under  $<$ ).
  - Downwardly closed sets correspond to sets of events that can form a prefix of a trace equivalent to  $\sigma$ .
- Note – label events to make them unique if required.

# How this works (2)

- We can also construct the FA as:
  - States are downwardly closed sets of event.
  - $\{\}$  is the initial state
  - The complete set of events is the final state.
  - There is a transition from set  $E$  to set  $E'$  with event  $e$  iff  $\{e\} = E' \setminus E$ .

- The number of states of the FA is the product of the lengths of the  $\sigma_i$  (plus 1)
- So, exponential space is required.
- However, polynomial time if  $m$  is bounded above.

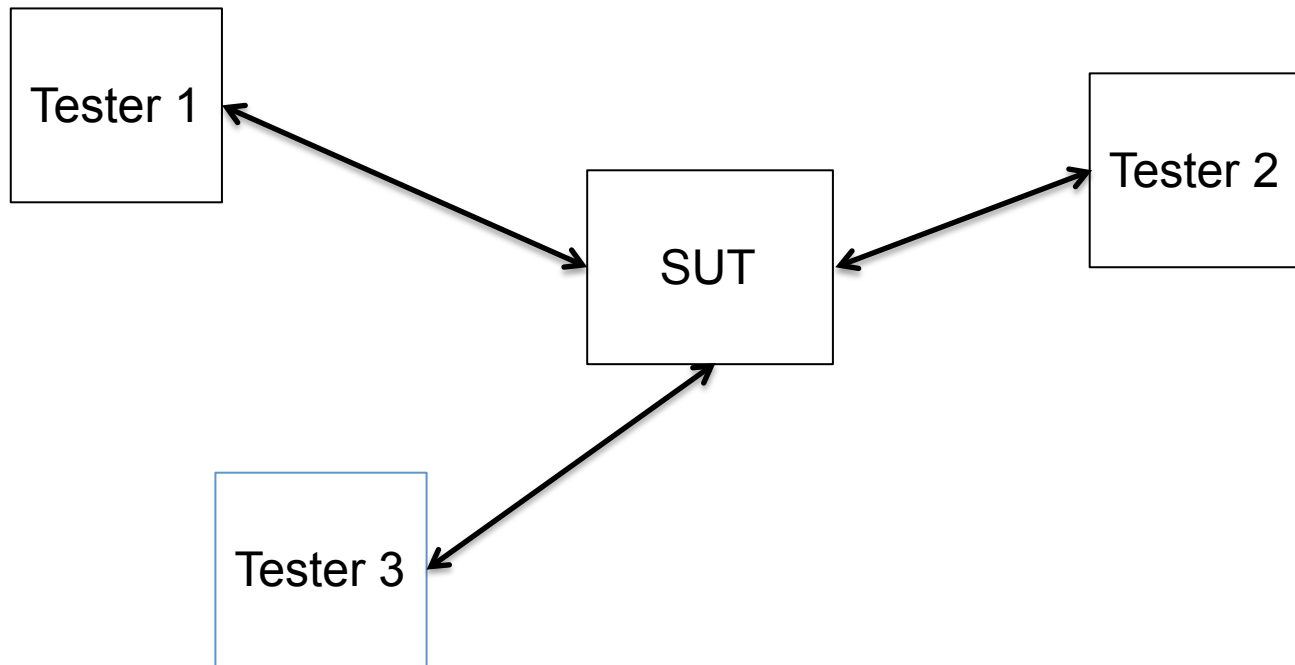
# Results

- For single port: Oracle Problem can be solved in low order polynomial time.
- For DFSMs in distributed testing:
  - Can be solved in polynomial time for controllable test sequences; otherwise NP-complete.
- For NFSMs:
  - NP-complete even for controllable testing.
- However, problems become polynomial if we place bounds on the number of ports.

# Distinguishing states and FSMs

- Let us suppose that:
  - $M$  is the specification
  - $N$  models a potential (and possibly faulty) implementation
- We want to know whether  $N$  conforms to  $M$ .
- Equivalently, we want to know whether two states can be distinguished.

# Independent testers



- We have separate, independent, testers.
- At any point:
  - The FSM being tested has a current state.
  - Each local tester has observed a local trace.
- There are infinitely many possible combinations of the above.

# Single port systems

- We can represent this as a two player game problem.
  - The state of the game is a pair of states (specification, implementation).
  - Tester moves: apply input
  - System moves: change state and return output.
- One player (the tester) wants to force the observation of a failure.

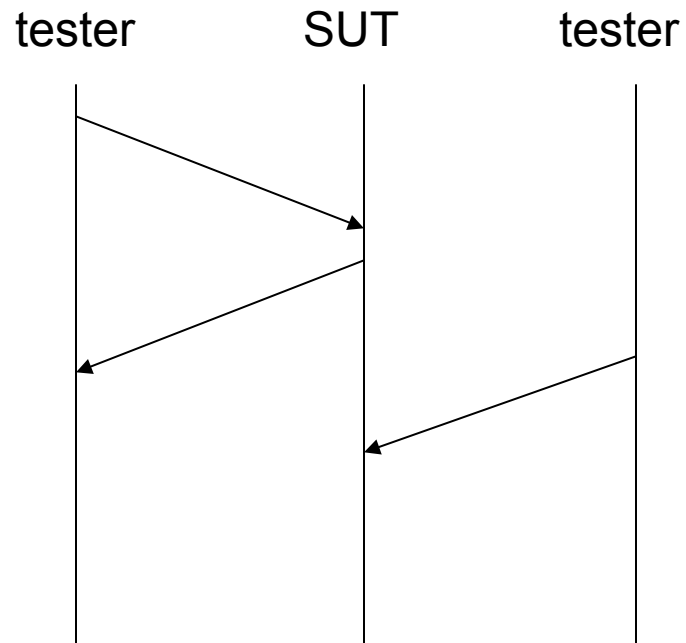


# Distinguishing FSMs: result

- Similar to a multi-player game problem.
- It is undecidable whether  $N$  conforms to  $M$  (and so also whether  $N$  is faulty).
- **Consequence:** there is no general algorithm for generating finite  $m$ -complete test suites for distributed testing.

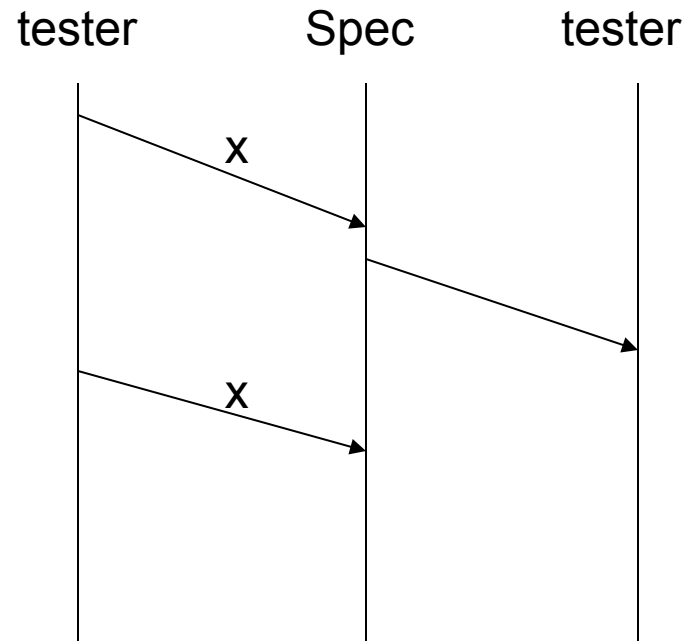
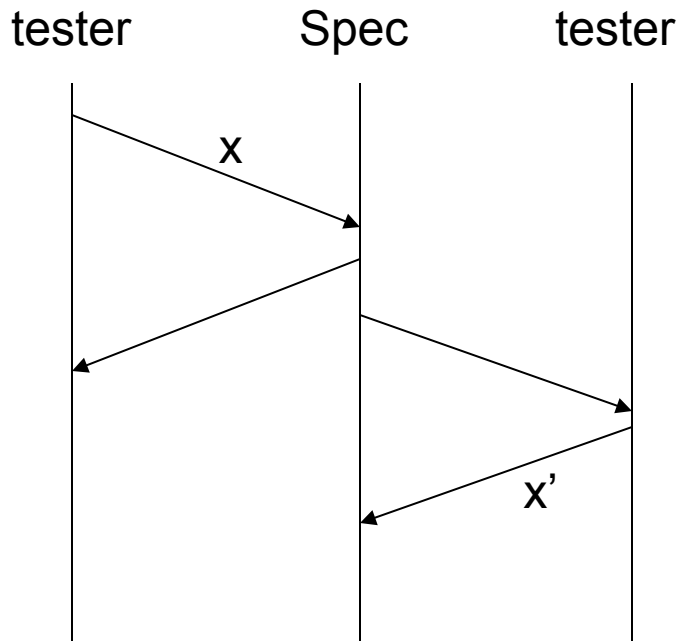
# Controllable testing

# This is not controllable



# Examples of controllability

- Two controllable scenarios



# What makes an input sequence controllable?

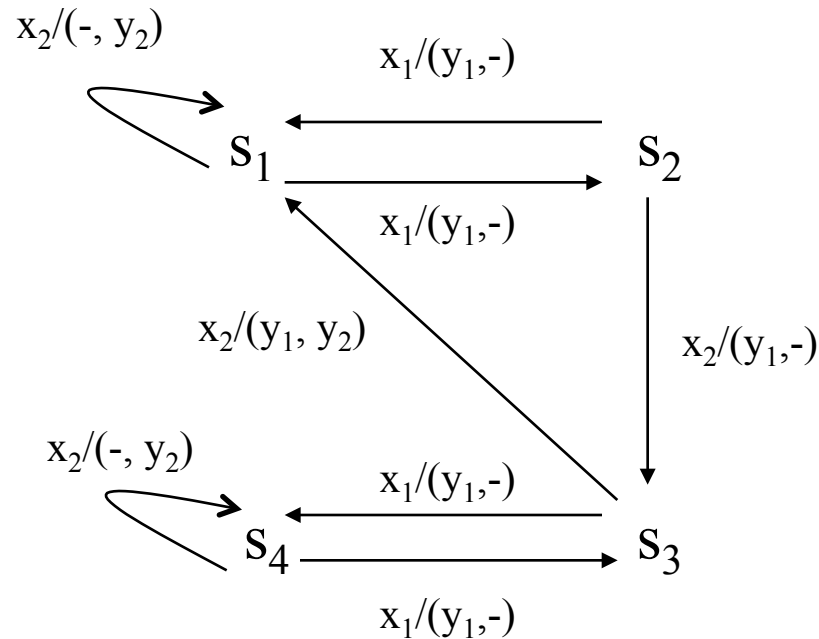
- In controllable testing:
  - We can follow the input of  $x$  in state  $s$  by input  $x'$  if:
    - $x$  and  $x'$  are at the same port; or
    - input  $x'$  is at a port  $p$  that receives output in response to  $x$ .
  - The first case relies on the atomicity of input/output pairs.

# Distinguishing states $s$ and $s'$

- If we restrict to controllable testing we need:
  - *(input sequence)  $x$  causes no controllability problems from  $s$  and  $s'$ .*
  - $x$  leads to different sequences of interactions, for  $s$  and  $s'$ , at *some port*.
- We say that  $x$  *locally  $s$ -distinguishes*  $s$  and  $s'$ .
- If no input sequence locally distinguishes  $s$  and  $s'$  they are *locally  $s$ -equivalent*.

# Testing is weaker

- We cannot locally s-distinguish  $s_1$  and  $s_4$  but  $x_1x_2$  can distinguish them.



# Distinguishing two states

- Given port  $p$  and states  $s_1$  and  $s_2$  of a  $k$ -port FSM  $M$  with  $n$  states:
  - $s_1$  and  $s_2$  are locally  $s$ -distinguishable by an input sequence starting at  $p$  if and only if they are locally  $s$ -distinguished by some such input sequence of length at most  $k(n-1)$ .
- This bound is tight.
- The sequences can be found in low-order polynomial time.



# Complete testing

- We know that:
  - There is no general algorithm for computing m-complete test suites.
  - There are benefits to using controllable test sequences.
- We might:
  - Try to achieve ‘as much as possible’ given that testing is controllable.

# $c(m)$ -complete test suites

- Given FSM  $M$  we say that test suite  $T$  is  $c(m)$ -complete if:
  - All test sequences in  $T$  are controllable.
  - For every FSM  $N$  with the same input/output alphabets as  $M$  and at most  $m$  states:
    - If  $N$  and  $M$  are locally  $s$ -distinguishable then some test sequence in  $T$  achieves this.
- i.e.  $T$  distinguishes between  $M$  and an SUT with at most  $m$  states *if this is possible in controllable distributed testing*.

# Generating $c(m)$ -complete test suites

# Restricting attention to controllable test sequences

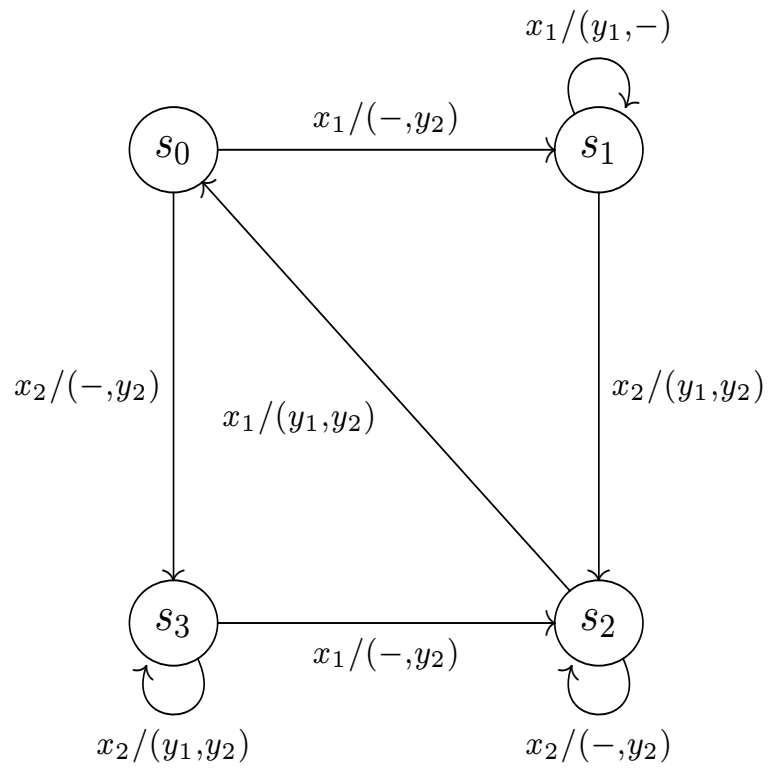
- We would like to represent the set of controllable test sequences.
- We will use a partial FSM to do this.

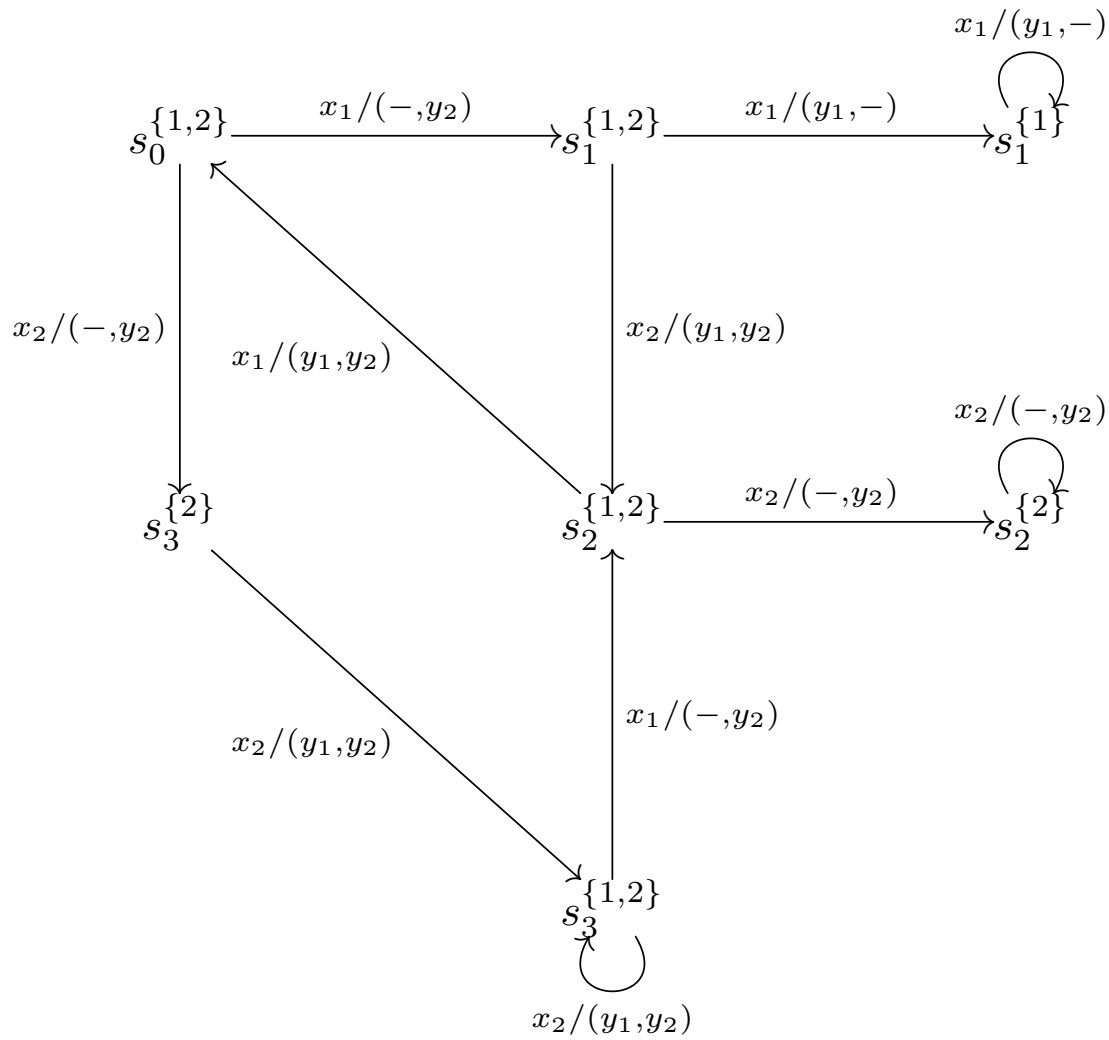
# Copies of states

- Let us suppose that:
  - $t$  is the transition  $(s',s,x/y)$ .
  - $P$  is the set of ports involved ( $p$  is in  $P$  if  $x$  is at  $p$  and/or  $y$  contains output at  $p$ ).
- We will represent the situation 'after  $t$ ' by state:  
 $s^P$
- The state  $s^P$  denotes the situation in which:
  - The FSM is in state  $s$  and can receive input at any port in  $P$  in controllable testing.

# Transitions leaving a 'new state'

- Let us suppose that:
  - t is the transition  $(s, s', x/y)$ .
- We will include a copy of t from every state of the form  $s^P$  such that:
  - Input x is at a port in P.
- We also include an initial state (initial state of M, input at any port).
- The combination defines the FSM  $M_{\min}$







# Results

- A path in  $M$  with label  $\sigma$  is controllable if and only if  $M_{\min}$  has a path with label  $\sigma$ .
- So:  $M_{\min}$  captures ‘controllable testing’

# Canonical FSMs

- Given FSM  $M$ , we can find:
  - Minimal  $M_{\min}$  that is locally s-equivalent to  $M$ .
  - Maximal (nondeterministic)  $M_{\max}$  that is locally s-equivalent to  $M$  (adding 'chaos state' to complete  $M_{\min}$ ).
- We can find them efficiently.

# Relevance of max and min machines

- Machine  $M_{\min}$  captures all of the traces that FSM  $N$  *has to implement* to conform to  $M$  (under s-equivalence).
- Machine  $M_{\max}$  contains all of the traces that an SUT *can have* without being distinguishable from  $M$  in controllable testing:
  - We can examine  $M_{\max}$  to determine whether it is acceptable to restrict attention to controllable test cases.

# Reaching states

- State  $s$  of  $M$  is reachable in controllable testing if and only if:
  - There is some  $P$  such that  $s^P$  is reachable in  $M_{\min}$
- Decidable in polynomial time.

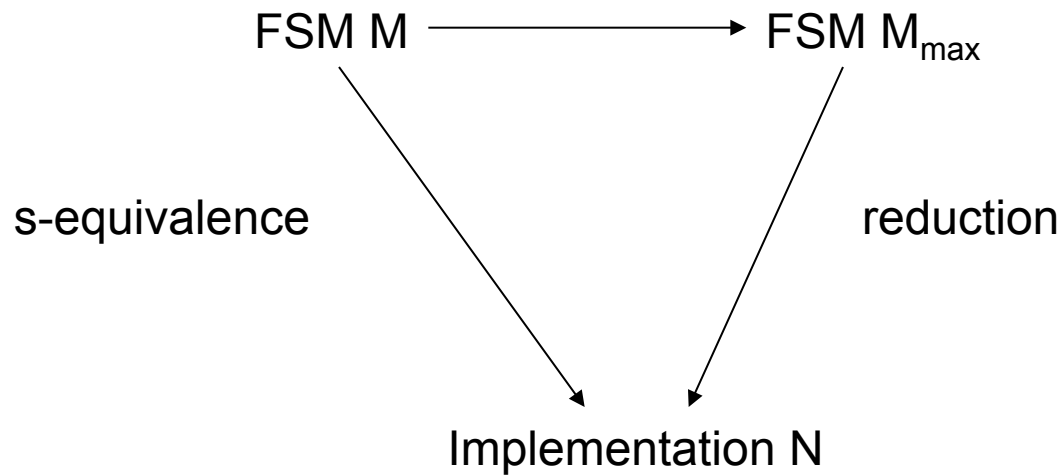
# Distinguishing states

- We have that  $s_1^P$  and  $s_2^{P'}$  are distinguishable in controllable testing if and only if:
  - There is a port  $p \in P \cap P'$  and input sequence  $x$  starting at  $p$  such that  $x$   $s$ -distinguishes  $s_1$  and  $s_2$ .
- Decidable in polynomial time.

# Comparing FSMs

- FSM  $N$  is locally s-equivalent to FSM  $M$  if and only if  $N_{\min}$  is equivalent to  $M_{\min}$ .
- FSM  $N$  is locally s-equivalent to FSM  $M$  if and only if  $N$  is a reduction of  $M_{\max}$ .
- Both decidable in polynomial time.

# Refinement and Testing



# Generating a $c(m)$ -complete test suite

- It is now straightforward:
  - We generate an  $m$ -complete test suite from partial FSM  $M_{\min}$ .
    - or
  - We generate an  $m$ -complete test suite from nondeterministic FSM  $M_{\max}$ .
- There are standard algorithms that can be adapted (e.g. using state counting).



# Efficiency issue

- Many test generation methods use:
  - Sets of pairwise distinguishable states.
- Size of test suite depends on how large these are.

# Graphs and cliques

- Given an undirected graph  $G=(V,E)$  we can generate an FSM  $M$  as follows:
  - Each vertex  $v_i$  in  $V$  is represented by a corresponding state  $s_i$  of  $M$ .
  - We can distinguish states  $s_i$  and  $s_j$  if and only if there is an edge between  $v_i$  and  $v_j$ .
- Consequence:
  - Finding a maximal set of pairwise distinguishable states of  $M$  is equivalent to finding a maximal clique of  $G$ .

# Consequence

- The problem of finding largest sets of pairwise distinguishable states is NP-hard.
- There are potential efficiency issues.
- Note:
  - This result also holds for single-port testing from a nondeterministic FSM or a partial FSM.

# Some papers (FSMs)

- B. Sarikara and G. Von Bochmann, Synthesis and Specification Issues in Protocol Testing, *IEEE Transactions on Communications*, **32** 4, pp. 389-395: 1984.
- R. Dssouli and G. von Bochmann. Error detection with multiple observers, *Protocol Specification, Testing and Verification V*, pp. 483-494: 1985.
- R. Dssouli and G. von Bochmann,. Conformance testing with multiple observers, *Protocol Specification, Testing and Verification VI*, pp. 217-229: 1986.
  
- R. M. Hierons and H. Ural. The effect of the distributed test architecture on the power of testing, *The Computer Journal*, **51** 4, pp. 497-510: 2008.
- R. M. Hierons: Canonical Finite State Machines for Distributed Systems, *Theoretical Computer Science*, **411** 2, pp. 566-580: 2010.
- R. M. Hierons: Verifying and Comparing Finite State Machines for Systems that have Distributed Interfaces, *IEEE Transactions on Computers*, **62** 8, pp. 1673-1683, 2013.
- R. M. Hierons: Oracles for Distributed Testing, *IEEE Transactions on Software Engineering*, **38** 3, pp. 629-641, 2012.
- R. M. Hierons: Generating Complete Controllable Test Suites for Distributed Testing, *IEEE Transactions on Software Engineering*, **41** 3, pp. 279-293 , 2015.
- R. M. Hierons and Uraz C. Turker: Distinguishing Sequences for Distributed Testing: Adaptive Distinguishing Sequences, *The Computer Journal* (to appear).

# Thanks

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# Conclusions

- If a system has distributed interfaces/ports then we have different implementation relations.
- This can affect testing and also development.
- We have new notions of correctness and corresponding test generation algorithms.
- Restricting attention to controllable test sequences brings practical benefits.

# Questions?