Testing Temporal Patterns

(Test Purpose Automata)

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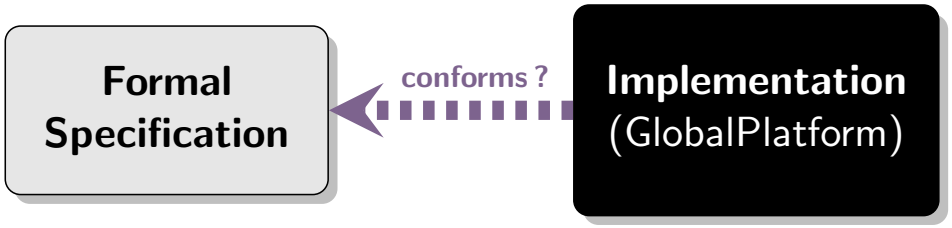
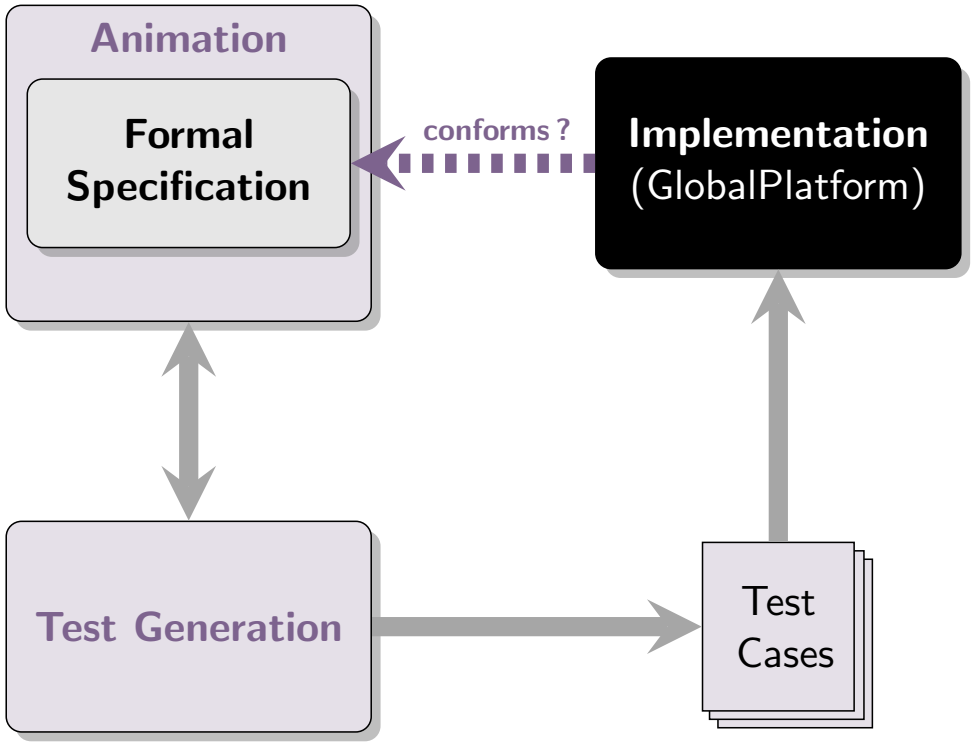


Context : Model-Based Testing

- Conformance testing : functional testing (black box)



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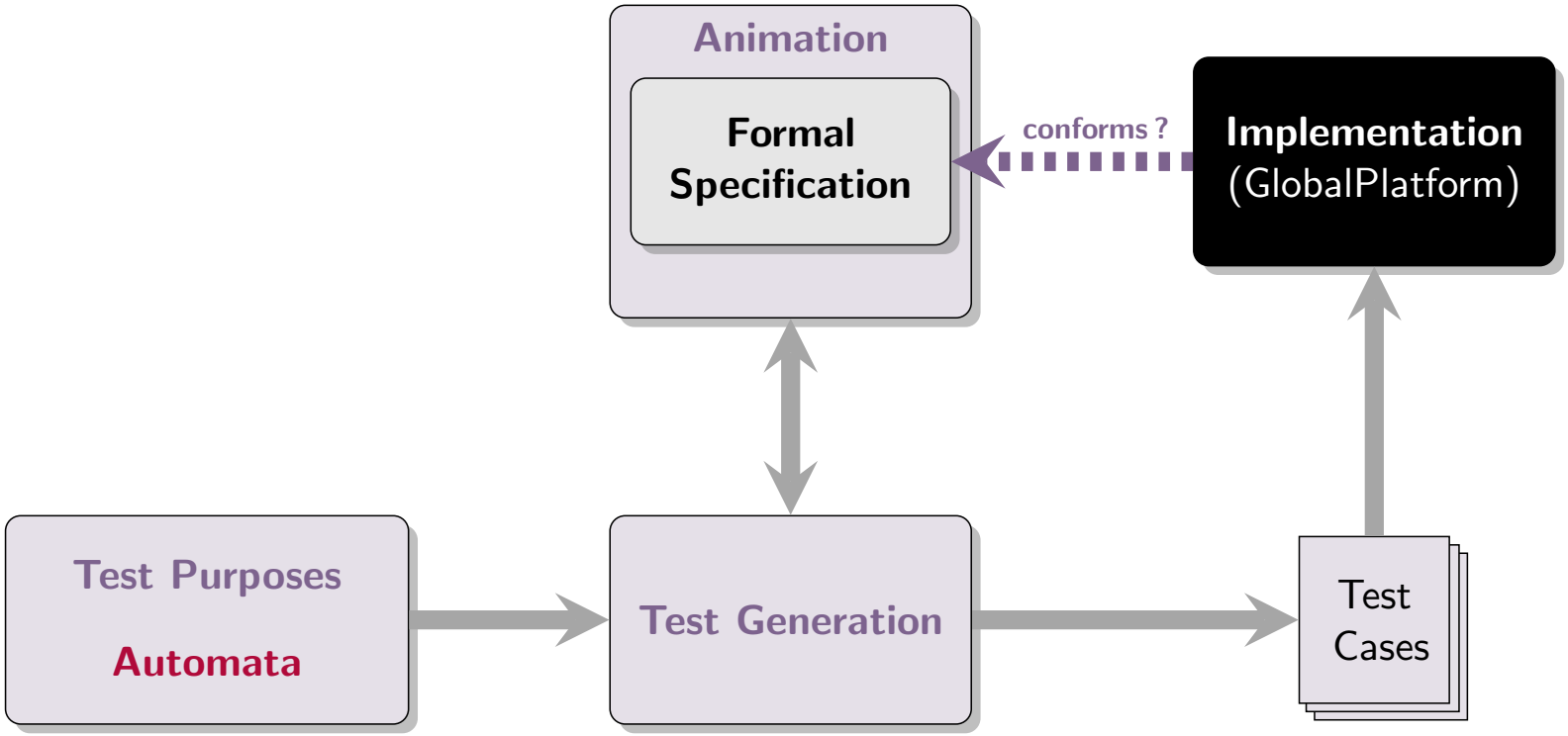
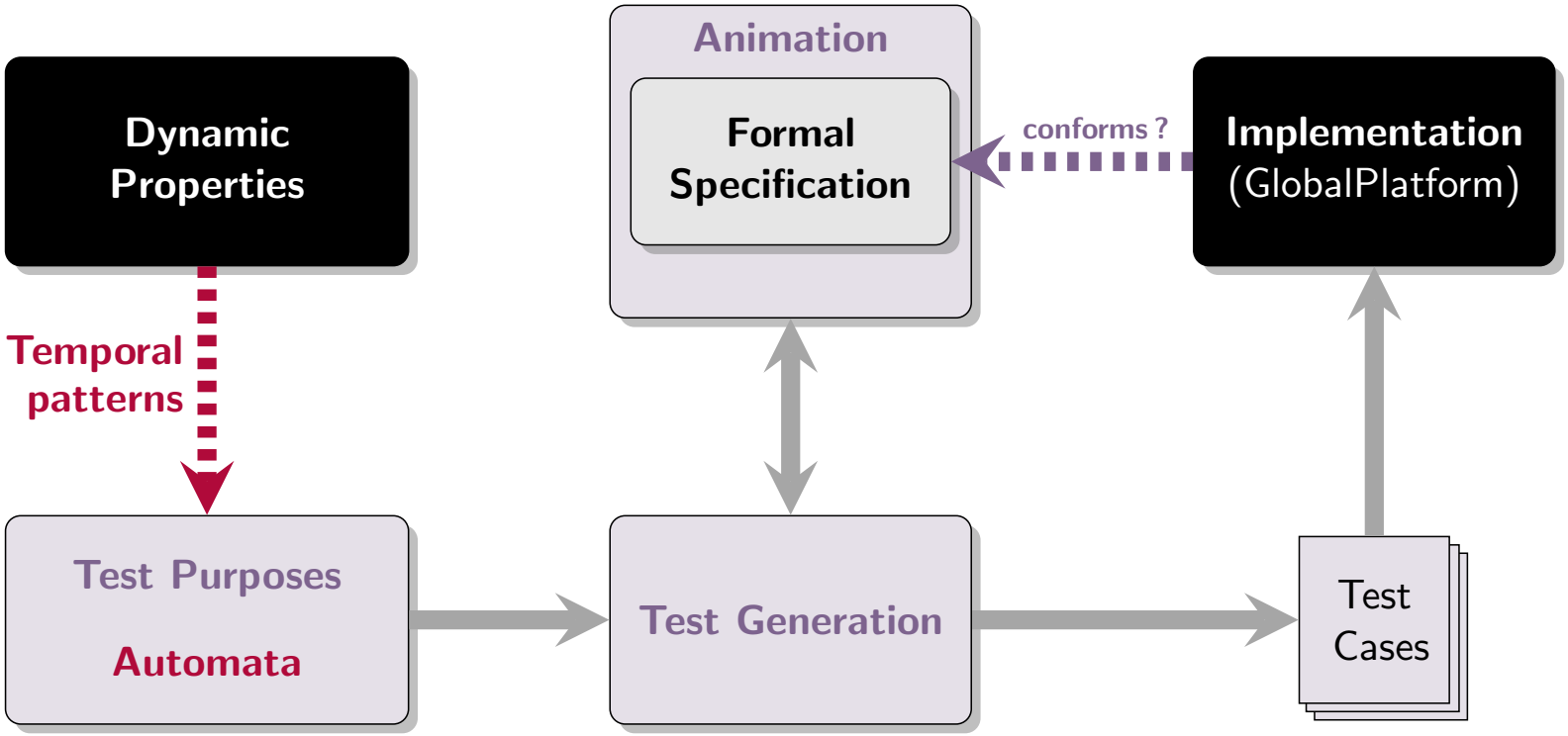
- Conformance testing : functional testing (black box)
- Generation/Animation of test cases



Context : Model-Based Testing

- Conformance testing : functional testing (black box)
- Generation/Animation of test cases
- Test Purposes automata



Objectives of Temporal patterns

1. Specify Dynamic Properties : Security, Safety, Liveness
2. Generate (automatically) Test Purposes Automata

Plan

Propositional ~~Linear Temporal~~ Logic (~~LTL~~)

$$\phi \quad ::= \quad true \quad | \quad a \quad | \quad \phi \quad \wedge \quad \phi \quad | \quad \neg \phi$$

where $a \in AP$

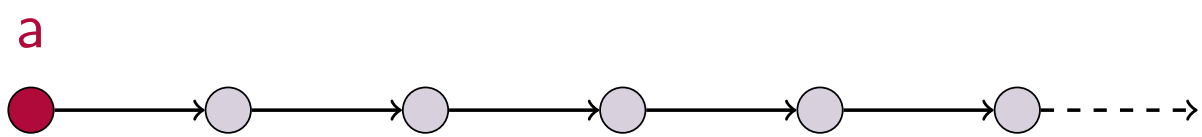
Propositional Linear Temporal Logic (LTL)

$$\phi \quad ::= \quad true \quad | \quad a \quad | \quad \phi \quad \wedge \quad \phi \quad | \quad \neg \phi \quad | \quad \bigcirc \phi$$

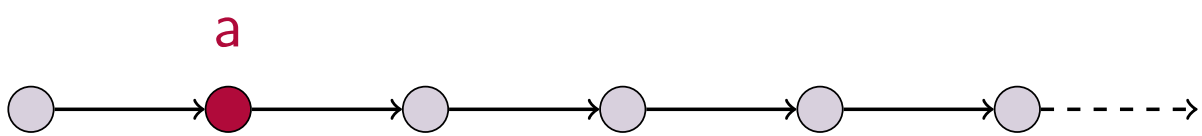
where $a \in AP$

$\bigcirc$ (next)

$\sigma \models a$



$\sigma \models \bigcirc a$



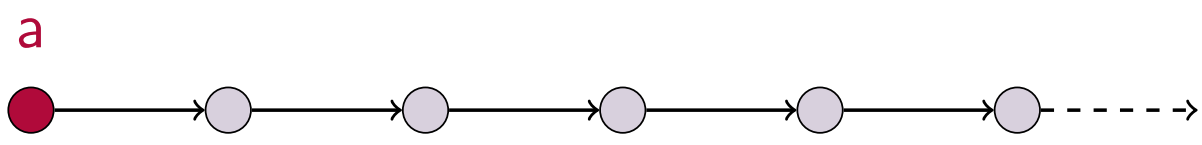
Propositional Linear Temporal Logic (LTL)

$$\phi \quad ::= \quad true \quad | \quad a \quad | \quad \phi \quad \wedge \quad \phi \quad | \quad \neg \phi \quad | \quad \bigcirc \phi \quad | \quad \Box \phi$$

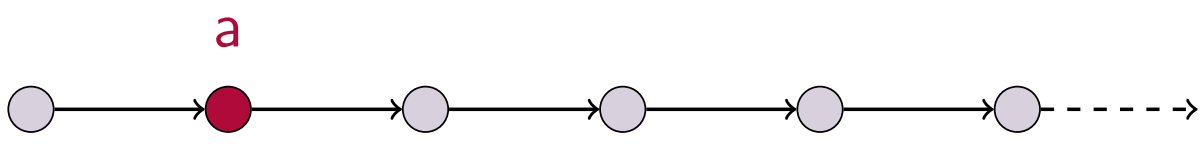
where $a \in AP$

$\bigcirc$ (next) $\Box$ (always)

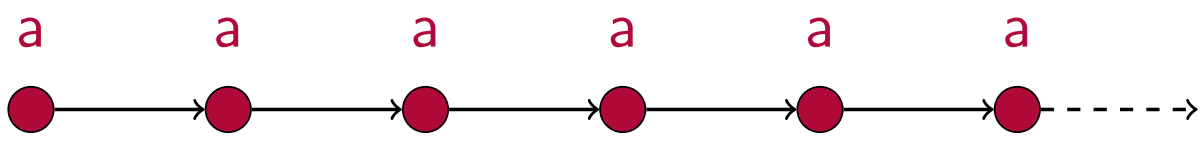
$\sigma \models a$



$\sigma \models \bigcirc a$



$\sigma \models \Box a$

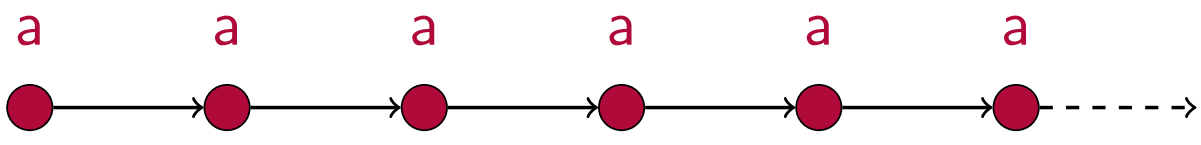


LTL : Derived Temporal Operators

$\Box\phi$

(always)

$\sigma \models \Box\ a$

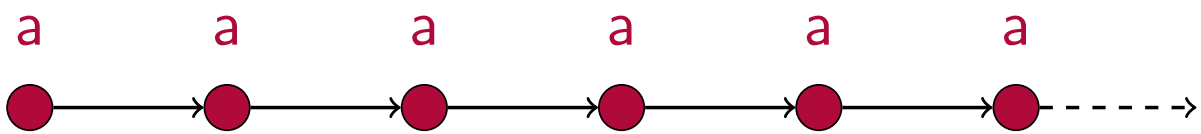


LTL : Derived Temporal Operators

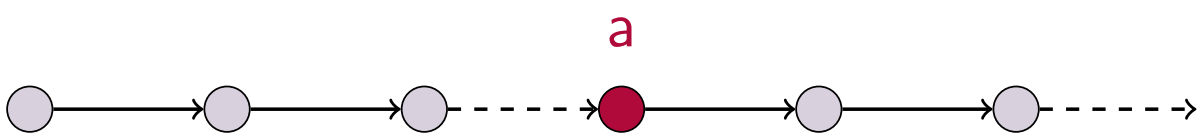
$\Box\phi$ (always)

$\Diamond\phi \equiv \neg\Box\neg\phi$ (eventually)

$\sigma \models \Box\ a$



$\sigma \models \Diamond\ a$



LTL : Derived Temporal Operators

$\Box\phi$ (always)

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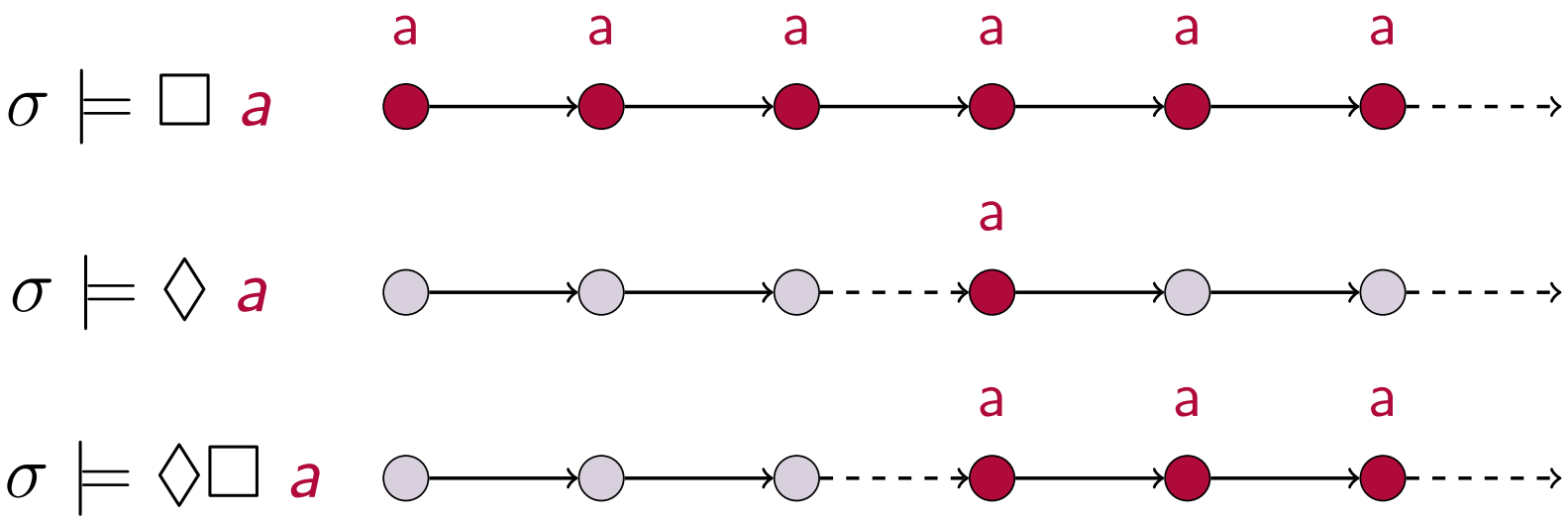
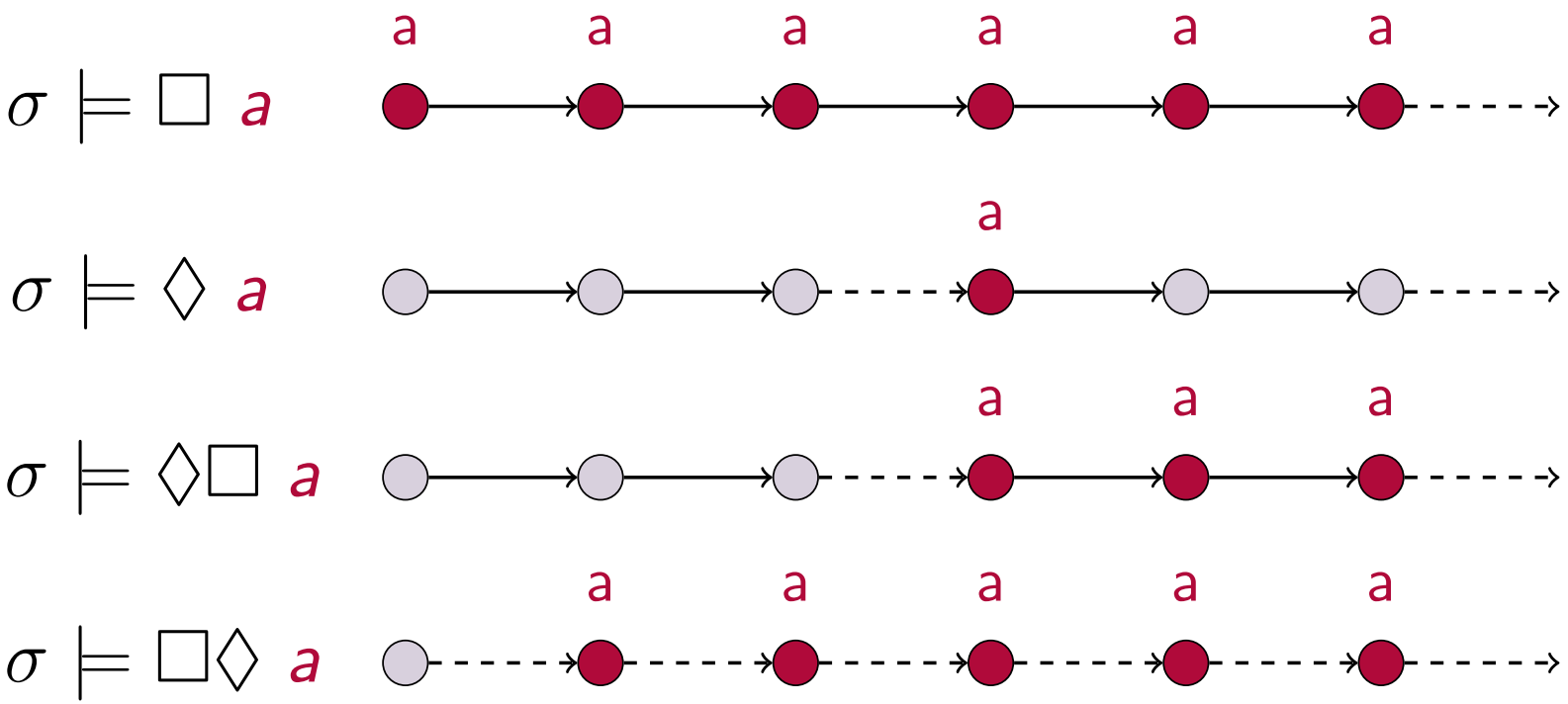
$\Diamond\Box\phi$ (persistence)



LTL : Derived Temporal Operators

$\Box\phi$ (always) $\quad \Diamond\phi \equiv \neg\Box\neg\phi$ (eventually) $\quad \Diamond\Box\phi$ (persistence) $\quad \Box\Diamond\phi \equiv \neg\Diamond\Box\neg\phi$ (infinitely many)



$\{\ i \mid a \in \sigma(i)\ \}$ is infinite

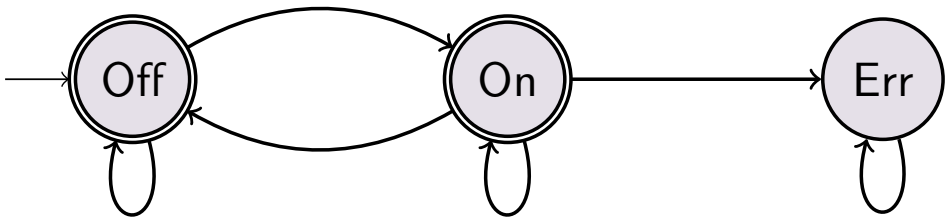
LTL : Example of Temporal Properties 1/2

- Safety :
  - cooling : $\Box\neg(temp_{high} \wedge cooling_{low})$
  - elevator : $\Box(moving \Rightarrow doors_{closed})$
  - traffic light : $\Box(yellow \Rightarrow \bigcirc red)$

LTL : Example of Temporal Properties 1/2

- Safety :
  - cooling : $\Box\neg(temp_{high} \wedge cooling_{low})$
  - elevator : $\Box(moving \Rightarrow doors_{closed})$
  - traffic light : $\Box(yellow \Rightarrow \bigcirc red)$
- Liveness :
  - progress : $\Diamond\ progress$
  - response : $\Box(try\_to\_send \Rightarrow \Diamond delivered)$
  - termination : $\Diamond\Box\ terminated$

LTL : properties of a trace



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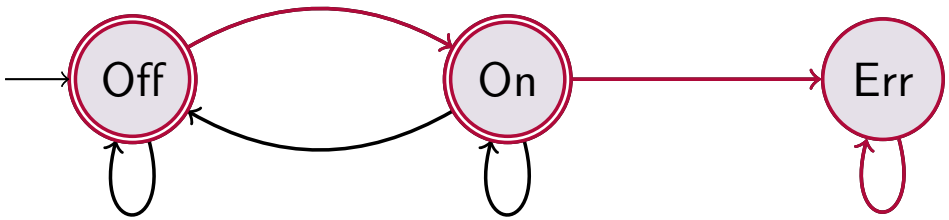
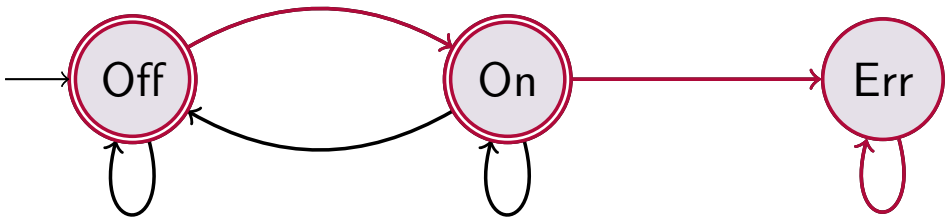


have a path $\pi = Off\ On\ Err\ Err\ Err \ldots = Off\ On\ Err^{\omega}$

- $\pi \models \bigcirc On$

LTL : properties of a trace



have a path $\pi = Off\ On\ Err\ Err\ Err \ldots = Off\ On\ Err^{\omega}$

- $\pi \models \bigcirc On$
- $\pi \models \bigcirc \bigcirc Err$

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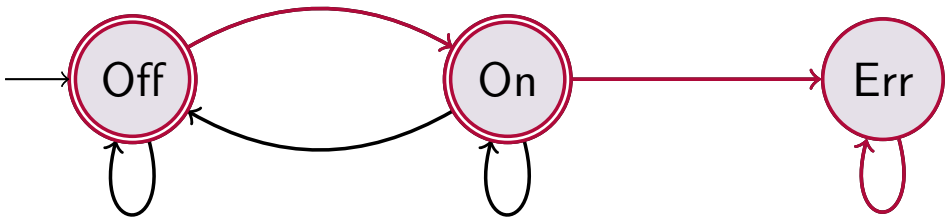

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- $\pi \models \bigcirc On$
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- $\pi \models (Off \vee On)\ \cup\ Err$

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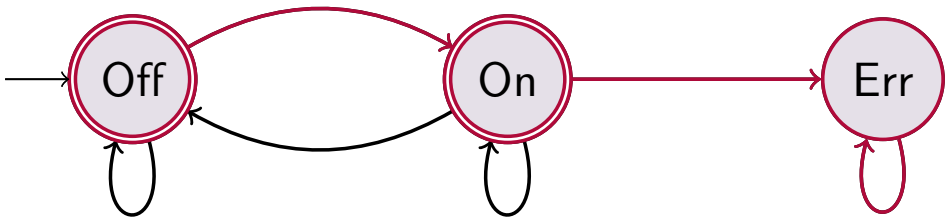

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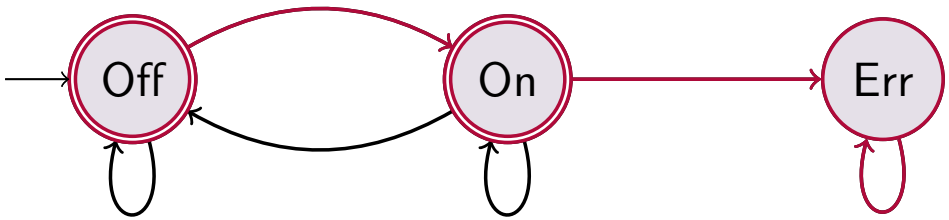
- $\pi \models \bigcirc On$
- $\pi \models \bigcirc \bigcirc Err$
- $\pi \models (Off \vee On)\ \cup\ Err$
- $\pi \models \Box(Err \Rightarrow \bigcirc Err)$

LTL : properties of a trace



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- $\pi \models \Box(Err \Rightarrow \Box Err)$

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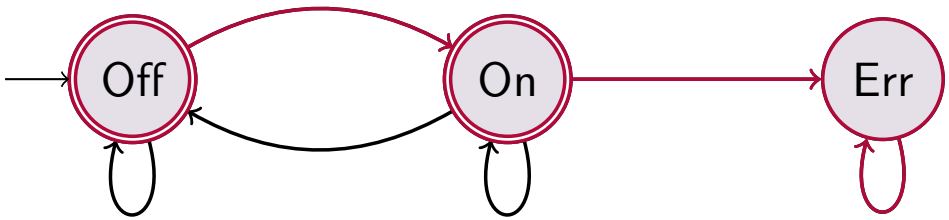

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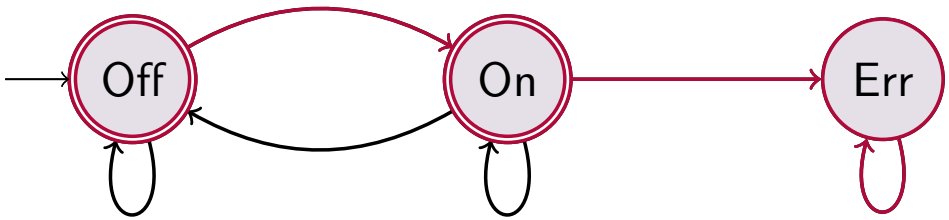
- $\pi \models \Box(Err \Rightarrow \Box Err)$
- $\pi \models \Diamond\Box Err$ $(persistence)$

LTL : properties of a trace



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- $\pi \models \Box(Err \Rightarrow \Box Err)$
- $\pi \models \Diamond\Box Err$ (*persistence*)
- $\pi \models \bigcirc \bigcirc \Box Err$

LTL : Laws

- negation :

$$\neg\Box\phi \equiv \Diamond \neg \phi, \qquad \neg\Diamond\phi \equiv \Box \neg \phi, \qquad \neg \bigcirc \phi \equiv \bigcirc \neg \phi$$

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Corollary

$\boxtimes_i \in \{\Box, \Diamond, \bigcirc\}$.

$$\neg \boxtimes_1 \boxtimes_2 \ldots \boxtimes_{n-1} \boxtimes_n \phi \equiv \overset{\curvearrowright}{\boxtimes}_1 \overset{\curvearrowright}{\boxtimes}_2 \ldots \overset{\curvearrowright}{\boxtimes}_{n-1} \overset{\curvearrowright}{\boxtimes}_n \neg \phi$$

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- next :

$$\Diamond \bigcirc \phi \overset{?}{\equiv} \bigcirc \Diamond\, \phi$$

LTL : Laws

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$$\neg \overset{\curvearrowright}{\Box}\, \phi \equiv \Diamond \neg\, \phi, \qquad \neg \overset{\curvearrowright}{\Diamond}\, \phi \equiv \Box \neg\, \phi, \qquad \neg \overset{\curvearrowright}{\bigcirc}\, \phi \equiv \bigcirc \neg\, \phi$$

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- next :

$$\Diamond \bigcirc\, \phi \equiv \bigcirc \Diamond\, \phi\,, \qquad \Box \bigcirc\, \phi \equiv \bigcirc \Box\, \phi$$

Corollary

$\boxtimes_i \in \{\Box, \Diamond, \neg\}$.

$$\boxtimes_1 \boxtimes_2 \ldots \boxtimes_{n-1} \boxtimes_n \bigcirc\, \phi \equiv \bigcirc\, \boxtimes_1 \boxtimes_2 \ldots \boxtimes_{n-1} \boxtimes_n\, \phi$$

LTL : Laws

$\Box\phi$	$\Diamond\phi \equiv \neg\Box\neg\phi$	$\Diamond\Box\phi$	$\Box\Diamond\phi \equiv \neg\Diamond\Box\neg\phi$
(always)	(eventually)	(persistence)	(infinitely many)

LTL : Laws

$\Box\phi$ (always) $\quad \Diamond\phi \equiv \neg\Box\neg\phi$ (eventually) $\quad \Diamond\Box\phi$ (persistence) $\quad \Box\Diamond\phi \equiv \neg\Diamond\Box\neg\phi$ (infinitely many)

- Idempotency : $\Box\Box\ \phi \stackrel{?}{\equiv} \Box\ \phi$

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- Absorption : $\Diamond\Box\Diamond\ \phi \stackrel{?}{\equiv} \Box\Diamond\ \phi$

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- Absorption : $\Diamond\Box\Diamond\ \phi \equiv \Box\Diamond\ \phi \qquad \Box\Diamond\Box\ \phi \equiv \Diamond\Box\ \phi$

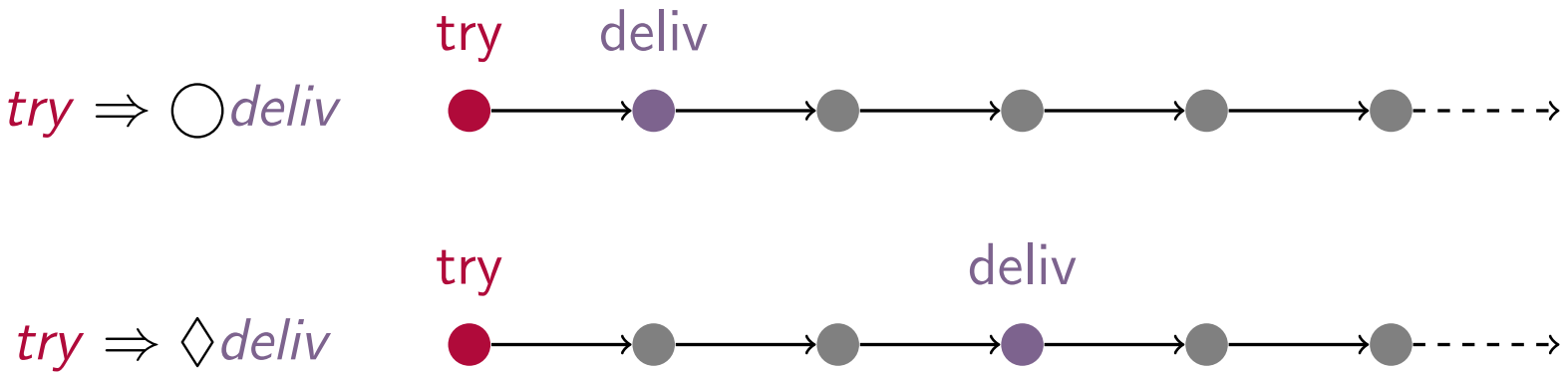
Corollary

$\boxtimes_i \in \{\Box, \Diamond\}$

$$\boxtimes_1 \boxtimes_2 \ldots \boxtimes_{n-1} \boxtimes_n\ \phi \equiv \begin{cases} \Box\ \phi & or \quad \Diamond\ \phi \quad or \\ \Box\Diamond\ \phi & or \quad \Diamond\Box\ \phi \end{cases}$$

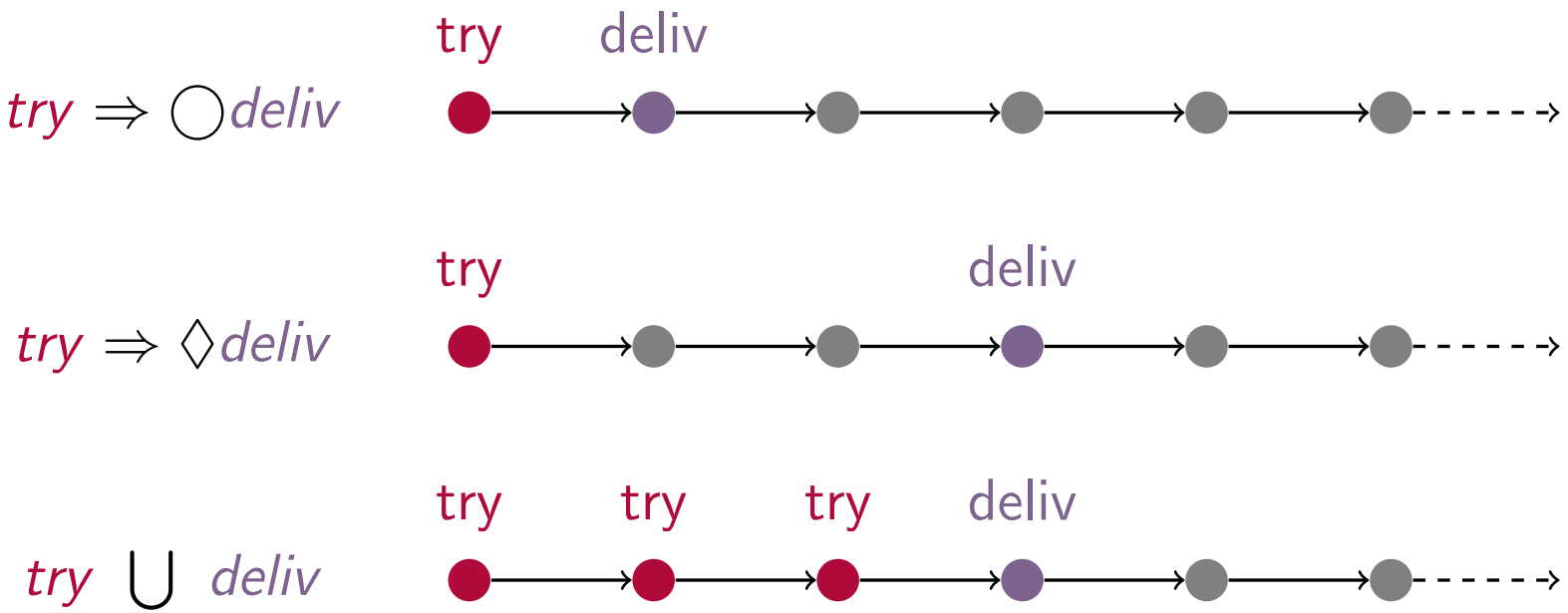
LTL : Until Operator

$$\phi \quad ::= \quad true \mid a \mid \phi \wedge \phi \mid \neg \phi \mid \bigcirc \phi \mid \Box \phi$$



LTL : Until Operator

$$\phi \ ::= \ true \mid a \mid \phi \ \wedge \ \phi \mid \neg \phi \mid \bigcirc \phi \mid \Box \phi \mid \phi \bigcup \phi$$



LTL : Until Operator

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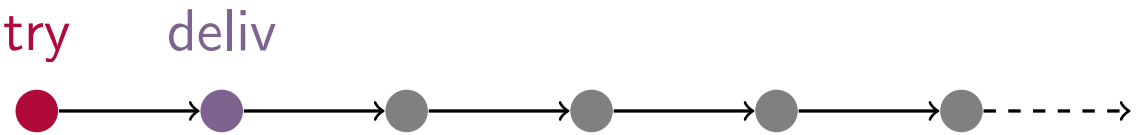
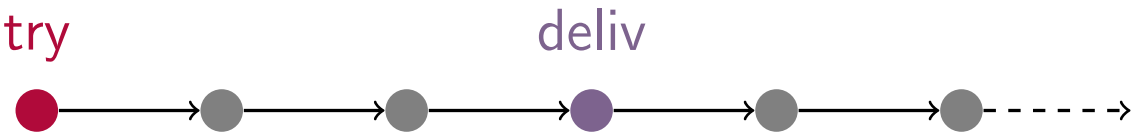
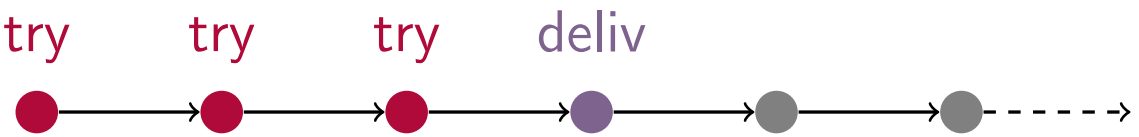
$try \Rightarrow \bigcirc deliv$



$try \Rightarrow \Diamond deliv$



$try \bigcup deliv$

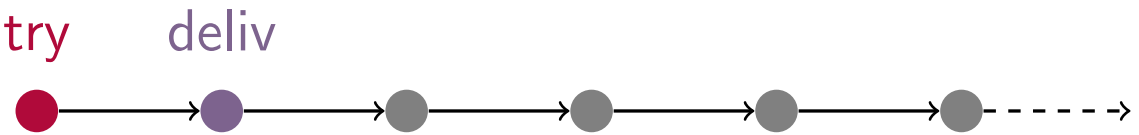


$$\Diamond \phi \equiv \mathbf{true} \cup \phi$$

LTL : Until Operator

$$\phi ::= true \mid a \mid \phi \wedge \phi \mid \neg \phi \mid \bigcirc \phi \mid \not\Box \phi \mid \phi \bigcup \phi$$

$try \Rightarrow \bigcirc deliv$



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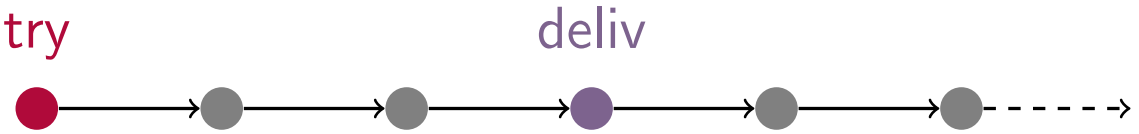

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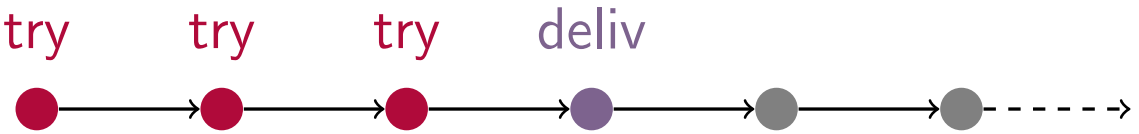

$\Diamond \phi \equiv \mathbf{true} \cup \phi$ and $\Box \phi \equiv \neg \Diamond \neg \phi$

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add the past-time dual of each temporal operator :

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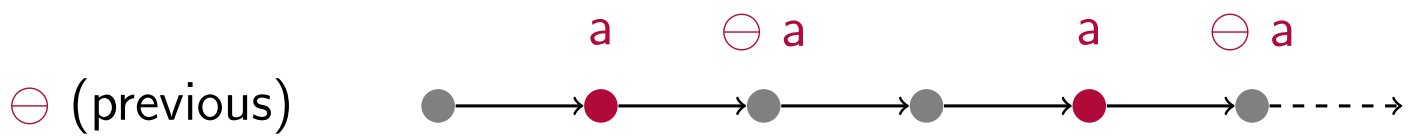

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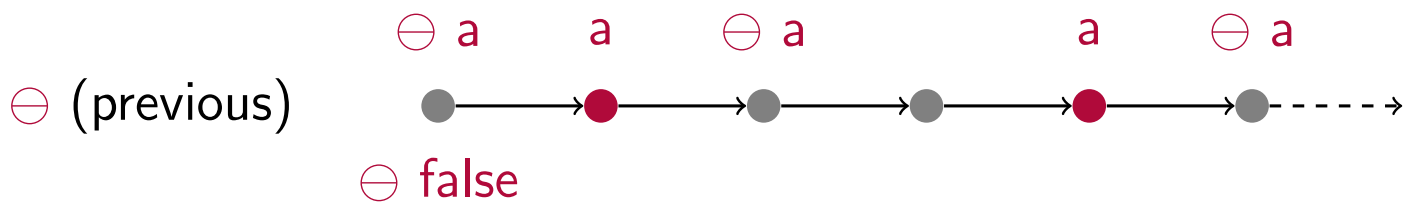

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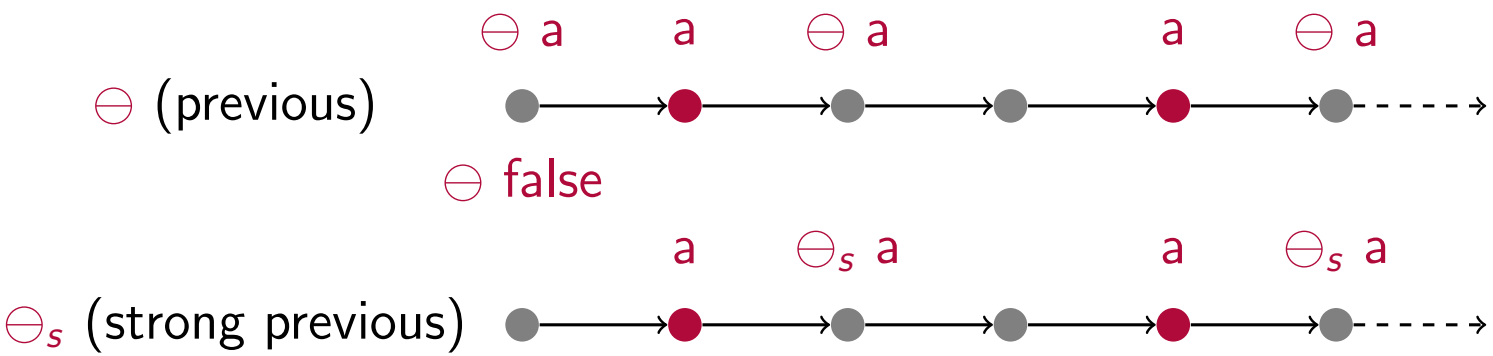

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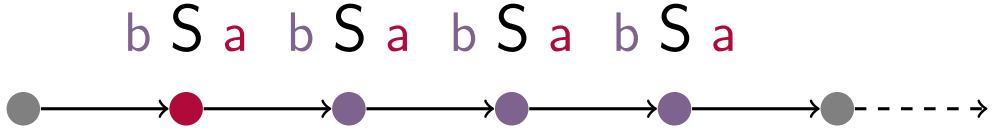

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LTL : Past Operators

Theorem **LTL + past $\stackrel{?}{\equiv}$ LTL** [Gabbay]

- an (LTL + past) formula can be written as as disjunction of 3 pure formulas :

$$\phi \equiv \underbrace{\phi_{past}}_{\mathsf{S},\ominus,\wedge,\neg} \vee \underbrace{\phi_{present}}_{\wedge,\neg} \vee \underbrace{\phi_{future}}_{\mathsf{U},\bigcirc,\wedge,\neg}$$

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Finite Automata (recall)

- A (Nondeterministic) Finite Automaton (NFA) $\mathcal{A}$ is a tuple $(\Sigma, S, I, \delta, F)$, where
  - Σ is a finite alphabet
  - S is a finite set of states
  - $I \subseteq S$ is the set of initial states
  - $\delta : S \times \Sigma \to 2^S$ is a transition function
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- A finite word $w = A_1 \dots A_n \in \Sigma^*$ is accepted by the NFA $\mathcal{A}$ if there exists a finite sequence $\pi = s_0 \dots s_n \in S^*$ such that :

$$s_0 \in I \quad \text{and} \quad \forall\, 0 \leq i < n,\ s_i \xrightarrow{A_{i+1}} s_{i+1} \quad \text{and} \quad s_n \in F$$

- We say that w belongs to the language of $\mathcal{A}$ noted $\mathcal{L}(\mathcal{A})$

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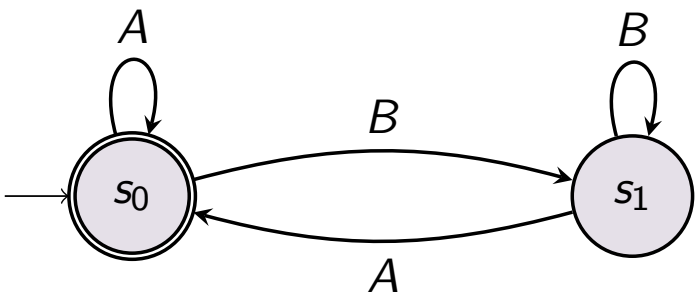
- we note $\mathcal{L}_\omega(\mathcal{A}) \subseteq \Sigma^\omega$ the accepted language of $\mathcal{A}$

NFA/NBA example

- Another example over the alphabet $\Sigma = \{A, B\}$



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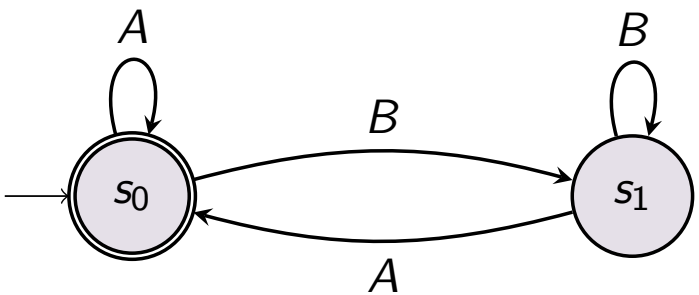

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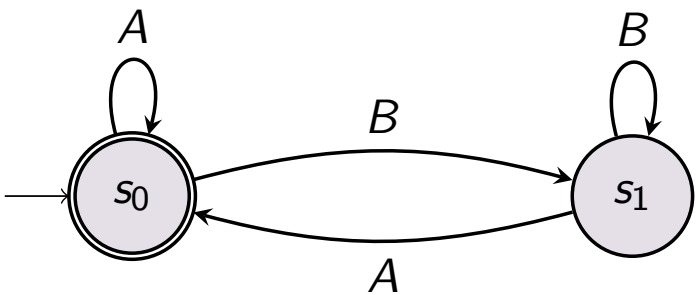

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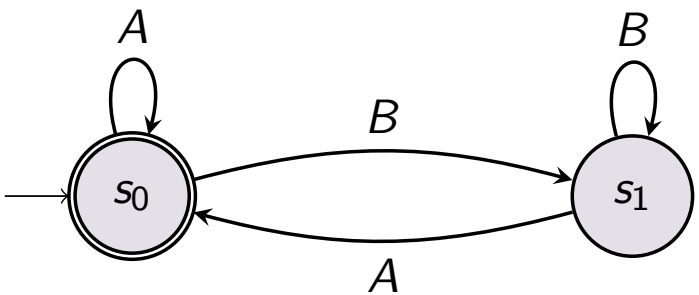

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$$\mathcal{L}_\omega(\mathcal{A}) \equiv ((A + B)^*.A)^\omega$$

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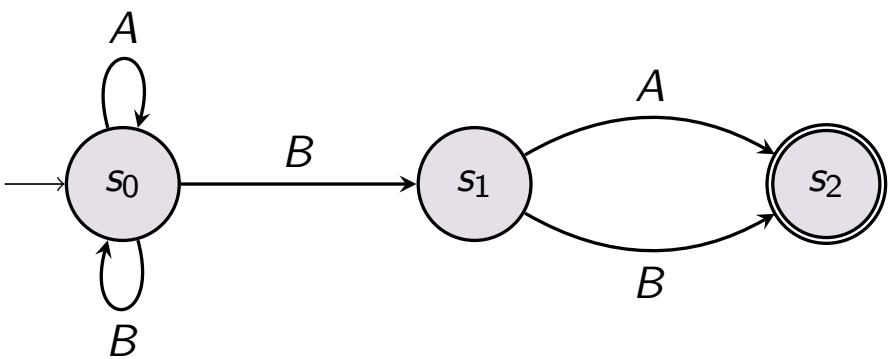

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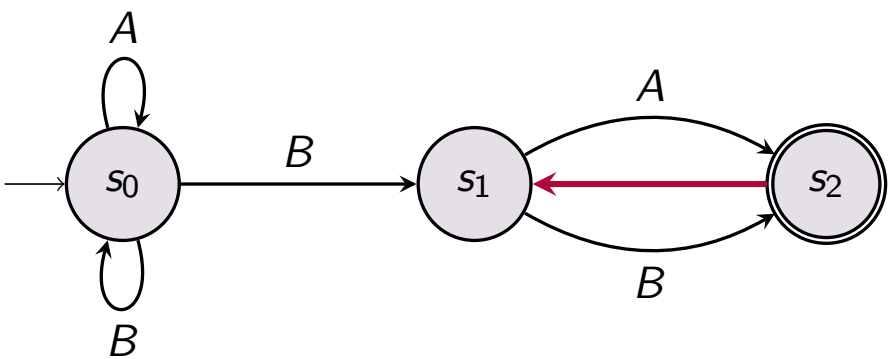

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Büchi Automata : Checking Emptiness

- Checking Emptiness principle :
  - If there is a reachable final state on a cycle $\Rightarrow \mathcal{L}_\omega(\mathcal{A}) \neq \emptyset$
- Checking Emptiness Algorithm : using a traversal algorithm
  1. starting from initial states, checking for the next reachable final state
  2. starting from such state, checking if it may reach itself (cycle)
  3. loop if no cycle
- Thus, checking Emptiness can be solved in linear time

from LTL to NBA

Theorem

For any Linear Time Property $P_\phi \subseteq (2^{AP})^\omega$ corresponding to some LTL-formula ϕ (over AP), there exists an NBA $\mathcal{A}_\phi$ over 2^{AP} such that :

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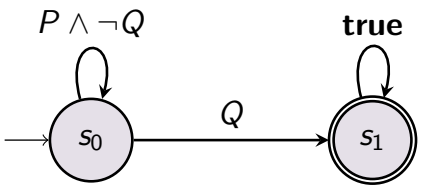

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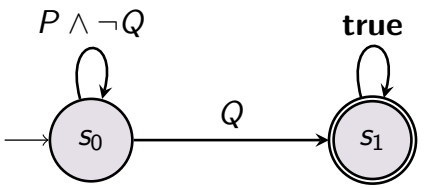

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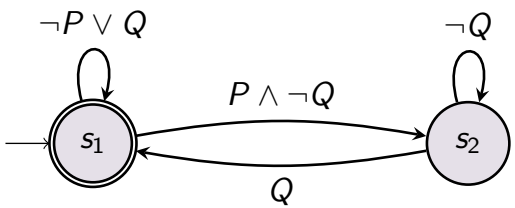

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    $subF(\phi)$?

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The GNBA $\mathcal{G}_\phi$ is a tuple $(\Sigma, S, I, \delta, \mathcal{F})$ where :

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$$\mathcal{B} \xrightarrow{A} \mathcal{B}' \in \delta \iff \begin{cases} A = \mathcal{B} \cap AP \\ \bigcirc \varphi \in \mathcal{B} \Leftrightarrow \varphi \in \mathcal{B}' \\ \varphi_1 \cup \varphi_2 \in \mathcal{B} \Leftrightarrow \varphi_2 \in \mathcal{B} \vee \\ \qquad (\varphi_1 \in \mathcal{B} \wedge \varphi_1 \cup \varphi_2 \in \mathcal{B}') \end{cases}$$

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- $\mathcal{F} = \{F_{\varphi_1 \cup \varphi_2} \mid \varphi_1 \cup \varphi_2 \in \mathit{subF}(\phi)\}$ where

$$F_{\varphi_1 \cup \varphi_2} = \{\mathcal{B} \in S \mid \varphi_1 \cup \varphi_2 \notin \mathcal{B} \vee \varphi_2 \in \mathcal{B}\}$$

Decidability of LTL

Theorem

SAT(LTL) is decidable

- Proof ?

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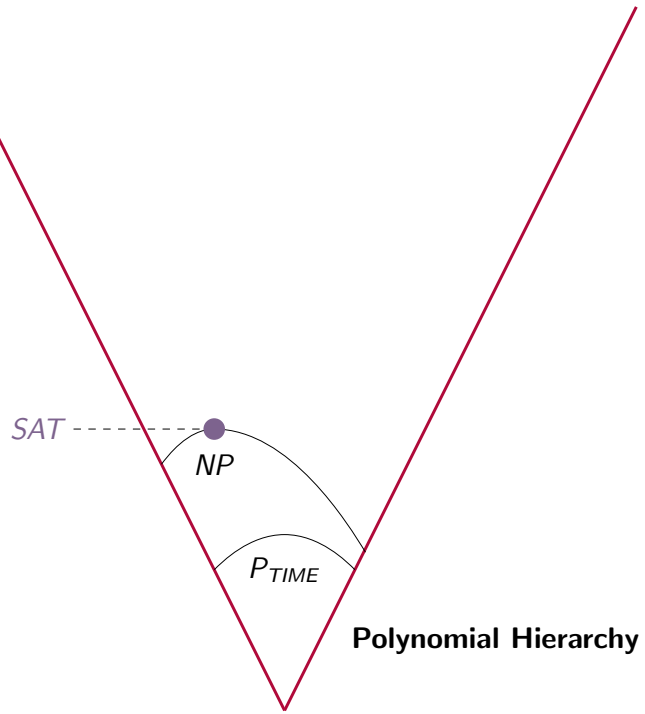
- VAL(LTL) is decidable (ex : LTL vs LTL)
- MC(LTL) is decidable (LTL vs Autmaton)

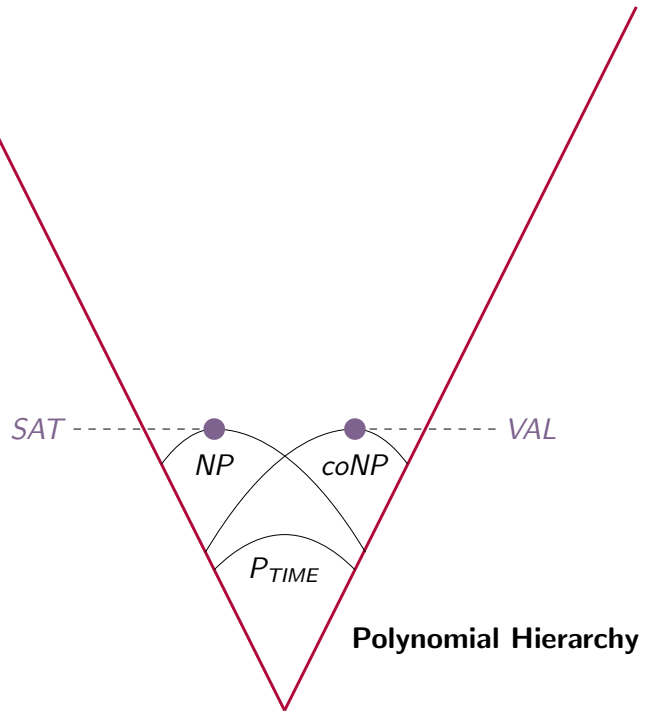
Complexity of LTL



Let $X_i = \{x_i^1, x_i^2, \ldots, x_i^n\}$ a vector of boolean variables,

The problem $SAT \equiv \exists X_1,\ Prop(X_1)$ is NP-complete

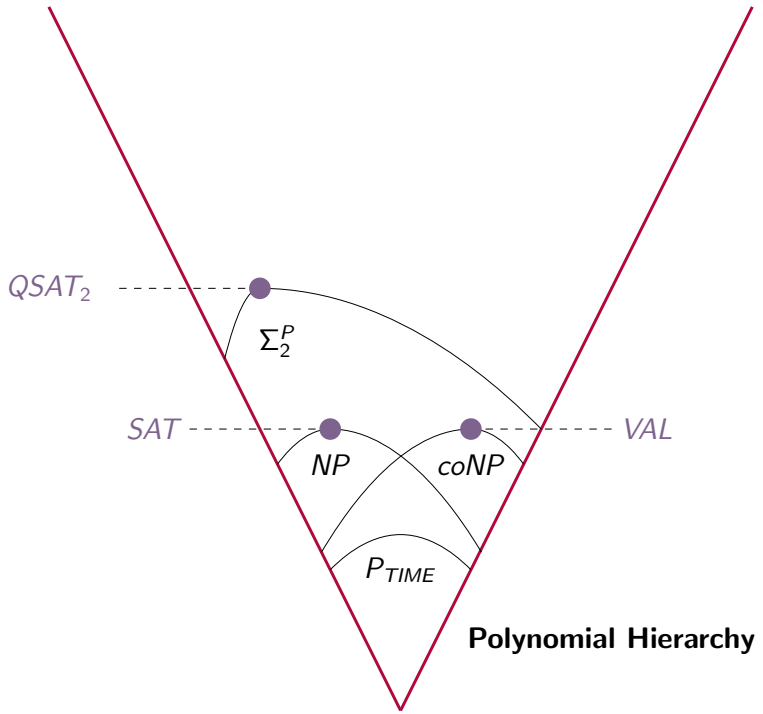
Complexity of LTL



Let $X_i = \{x_i^1, x_i^2, \ldots, x_i^n\}$ a vector of boolean variables,

The problem $VAL \equiv \forall X_1,\ Prop(X_1)$ is **co**NP-complete

Complexity of LTL



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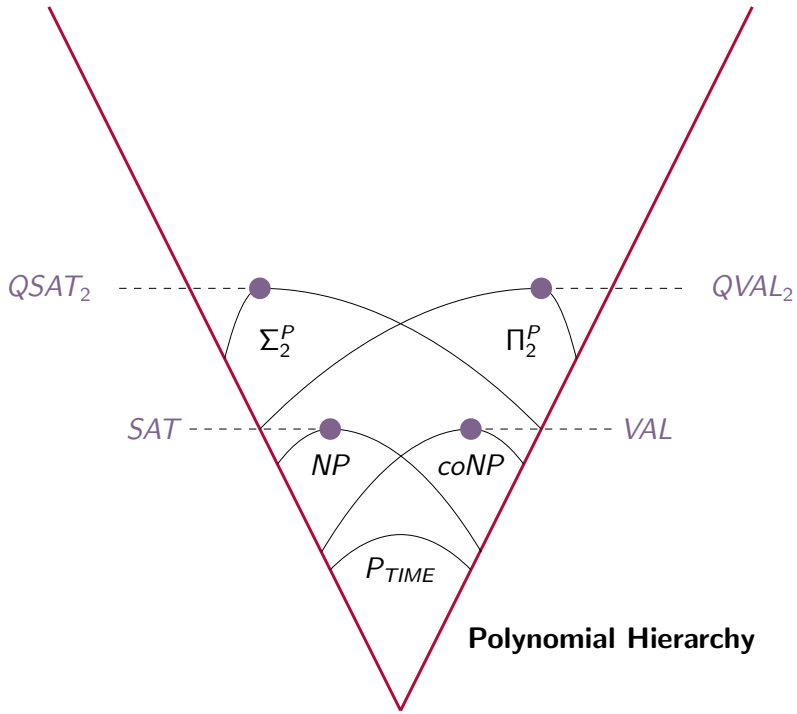

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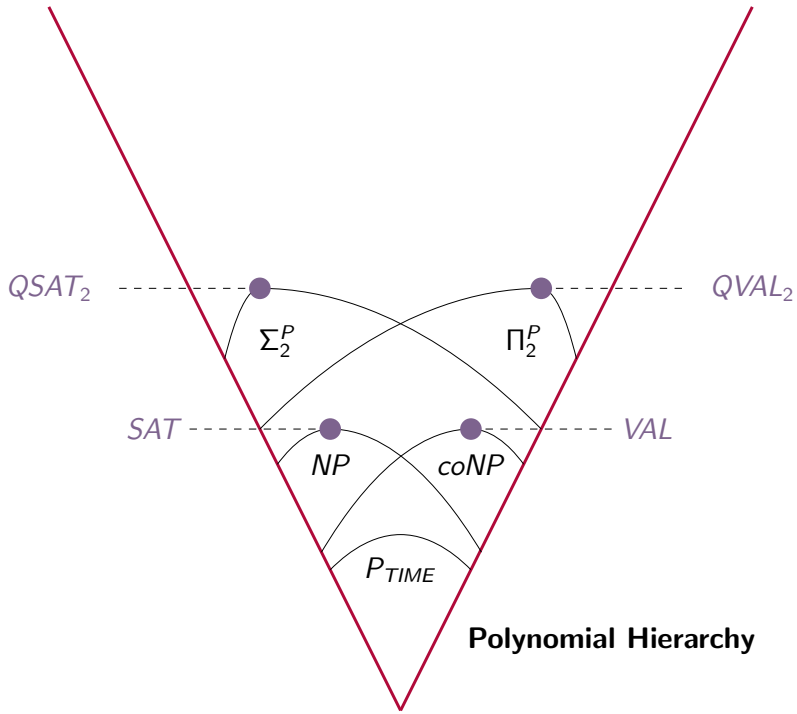

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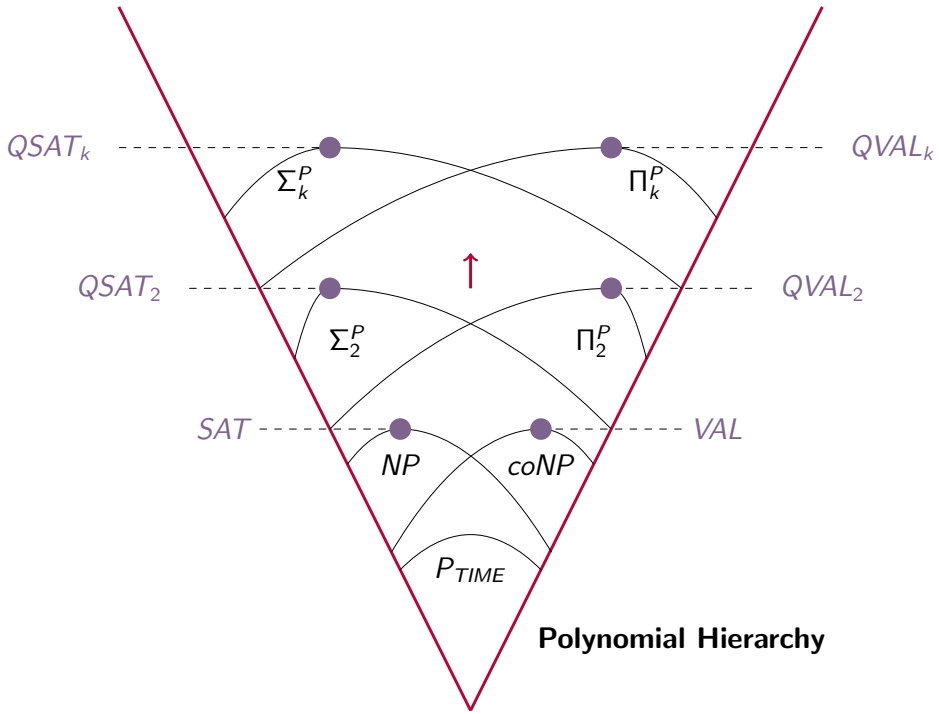
$$\Sigma_2^P \equiv NP^{coNP} \qquad \Pi_2^P \equiv coNP^{NP}$$

Complexity of LTL



The problem $QSAT_k \equiv \exists X_k, \forall X_{k-1} \ldots, \; Prop(X_1, \ldots, X_k)$ is Σ_k^P-complete

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$$\Sigma_k^P \equiv NP^{\Pi_{k-1}^P} \qquad \Pi_k^P \equiv coNP^{\Sigma_{k-1}^P}$$

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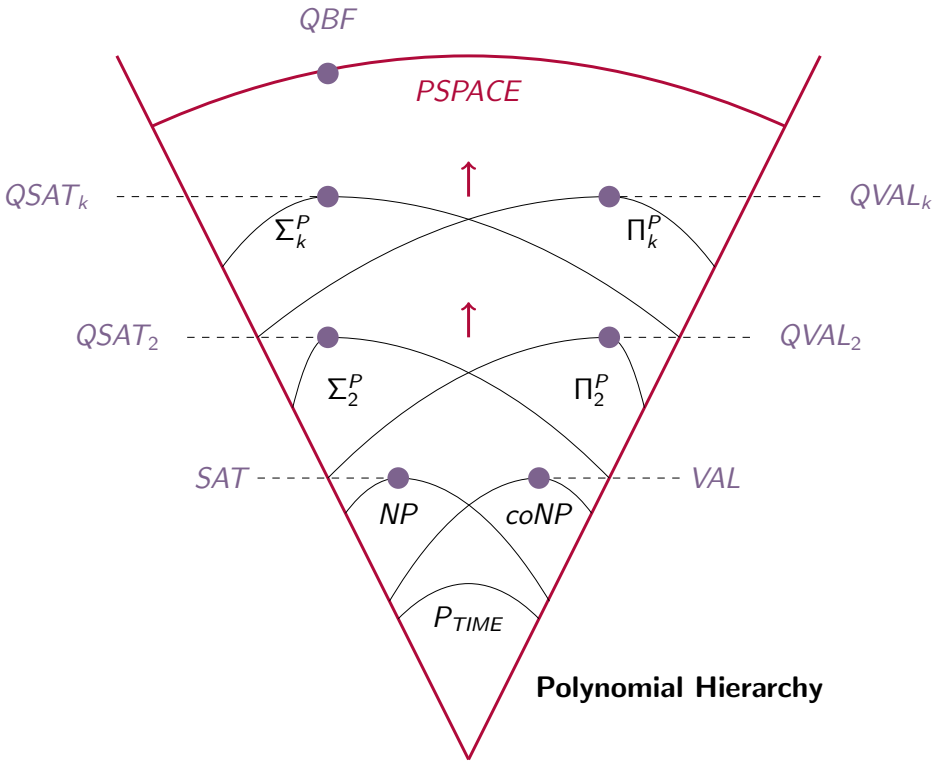
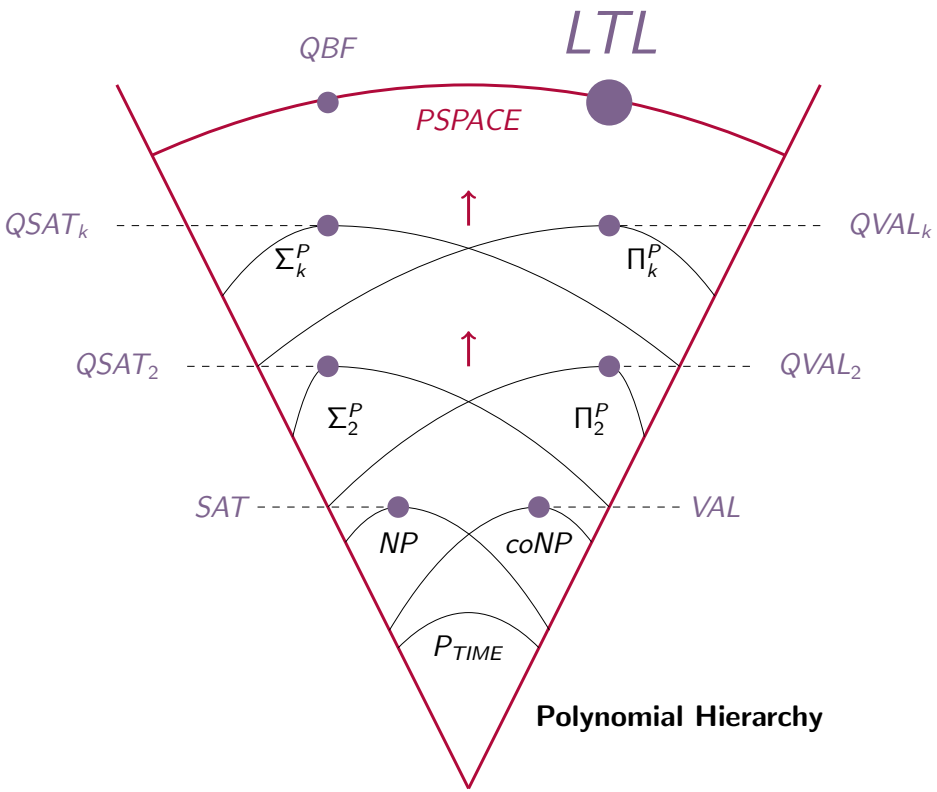


Let $Q_i \in \{\forall, \exists\}$, the general problem :

$QBF \equiv Q_n x_n, Q_{n-1} x_{n-1} \ldots, Q_1 x_1,\ Prop(x_n, \ldots, x_1)$ is $PSPACE$-complete

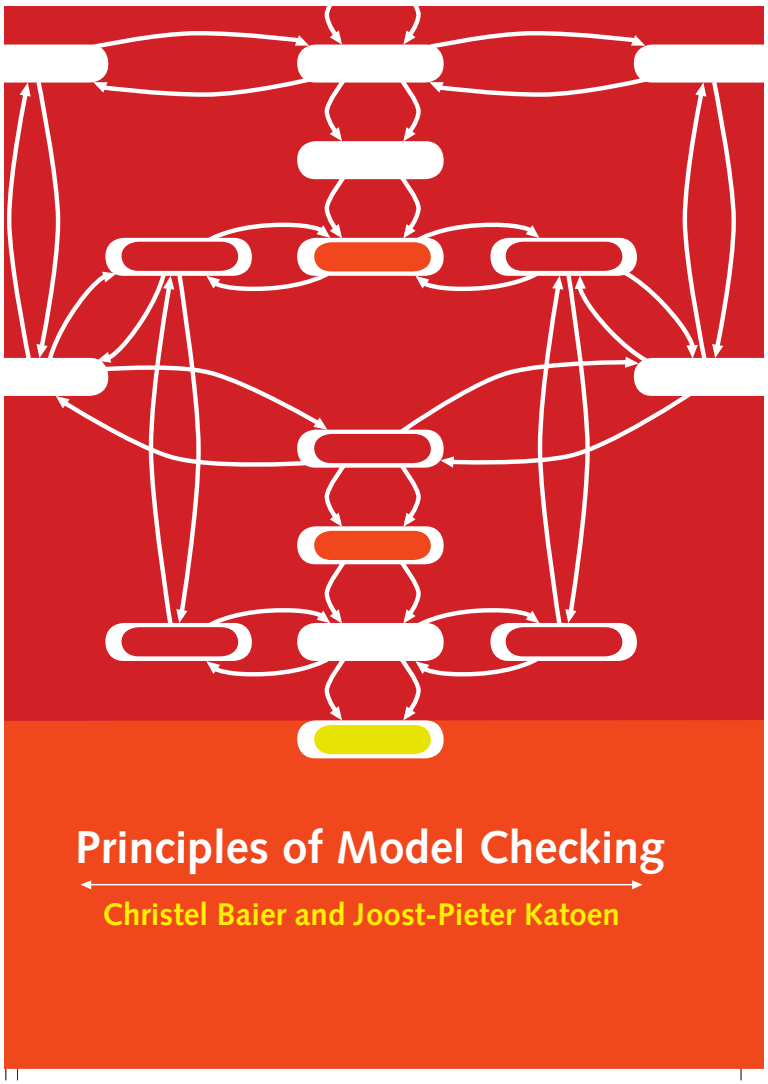
$$PSPACE \equiv NPSPACE \equiv coPSPACE$$

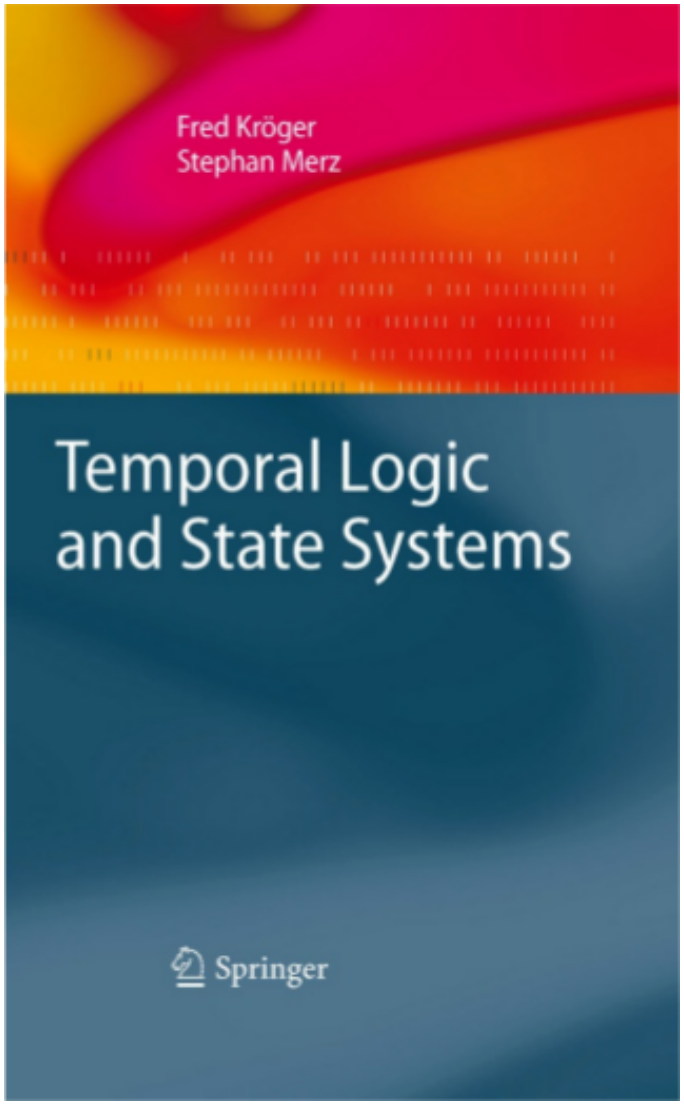
Complexity of LTL



The problems $SAT(LTL) \equiv VAL(LTL) \equiv MC(LTL)$ are *PSPACE*-complete

References


Principles of Model Checking
Christel Baier and Joost-Pieter Katoen


Fred Kröger
Stephan Merz
Temporal Logic
and State Systems
Springer

Plan

Context : Temporal Logics

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Issue

bridging the semantic gap between the natural language and Temporal Logics (the formal language)

Principle

- Can we provide formal semantics (of Temporal Logics) to the natural language ?

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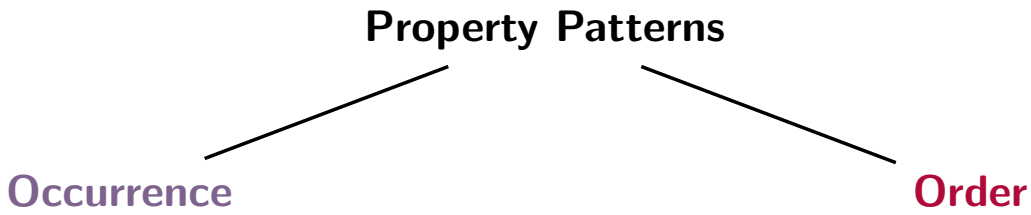
Dwyer et al.'s Temporal Patterns [Dwyer et al., 1999]

- Dwyer et al. predefine many patterns of temporal properties
- Each temporal property is a combination of :

Pattern / Scope

Patterns

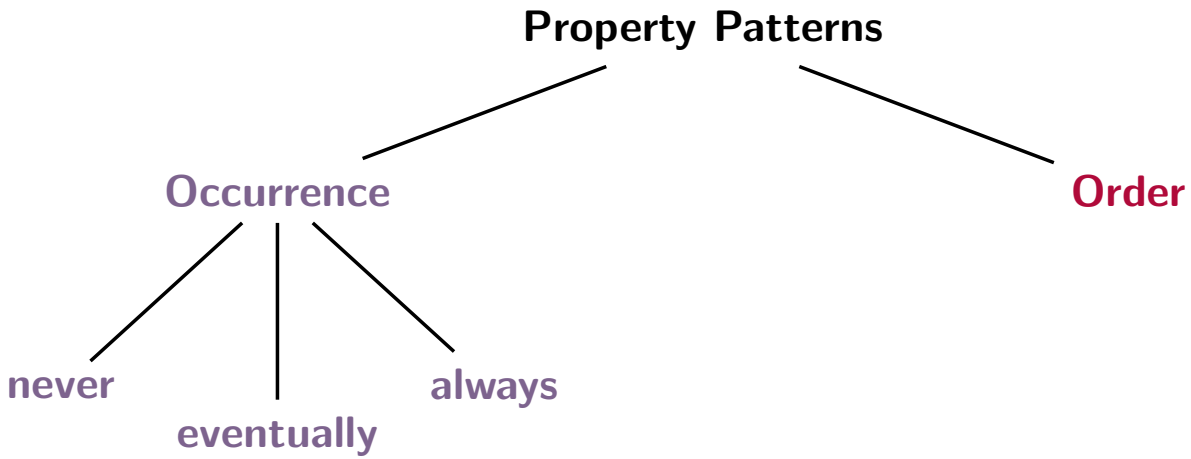


Property patterns

Occurrence patterns : Absence/Presence of events

Order patterns : Order between events

Patterns



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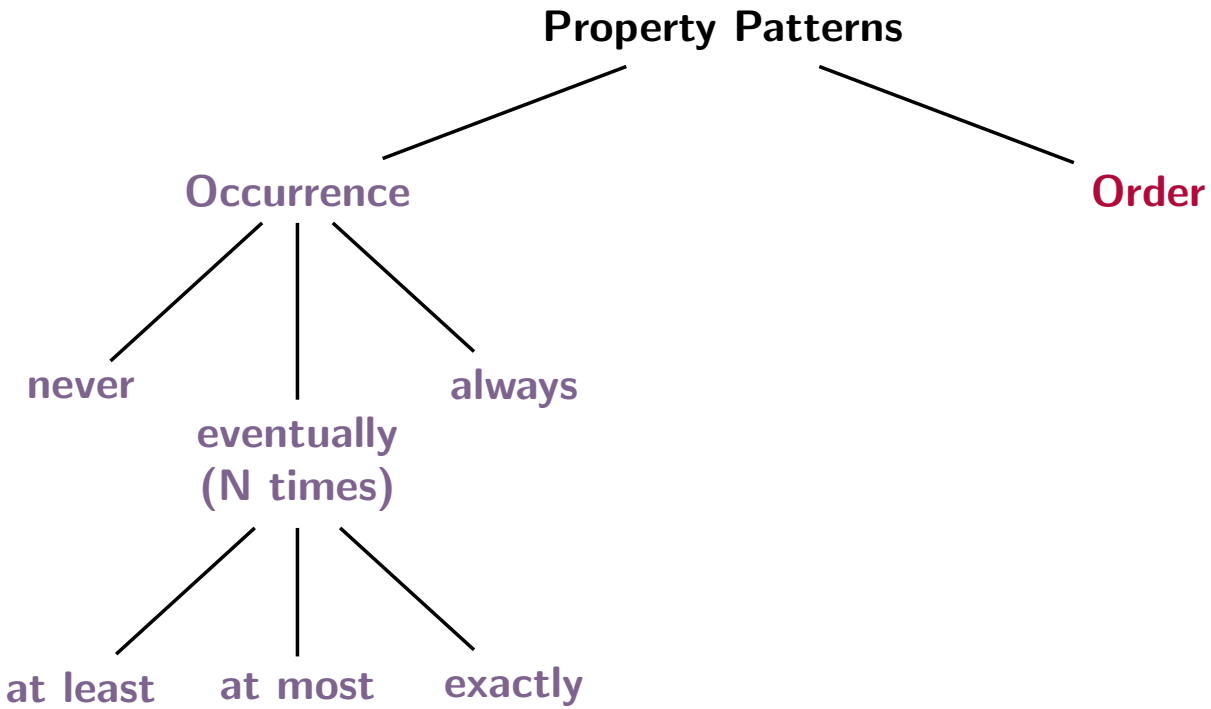

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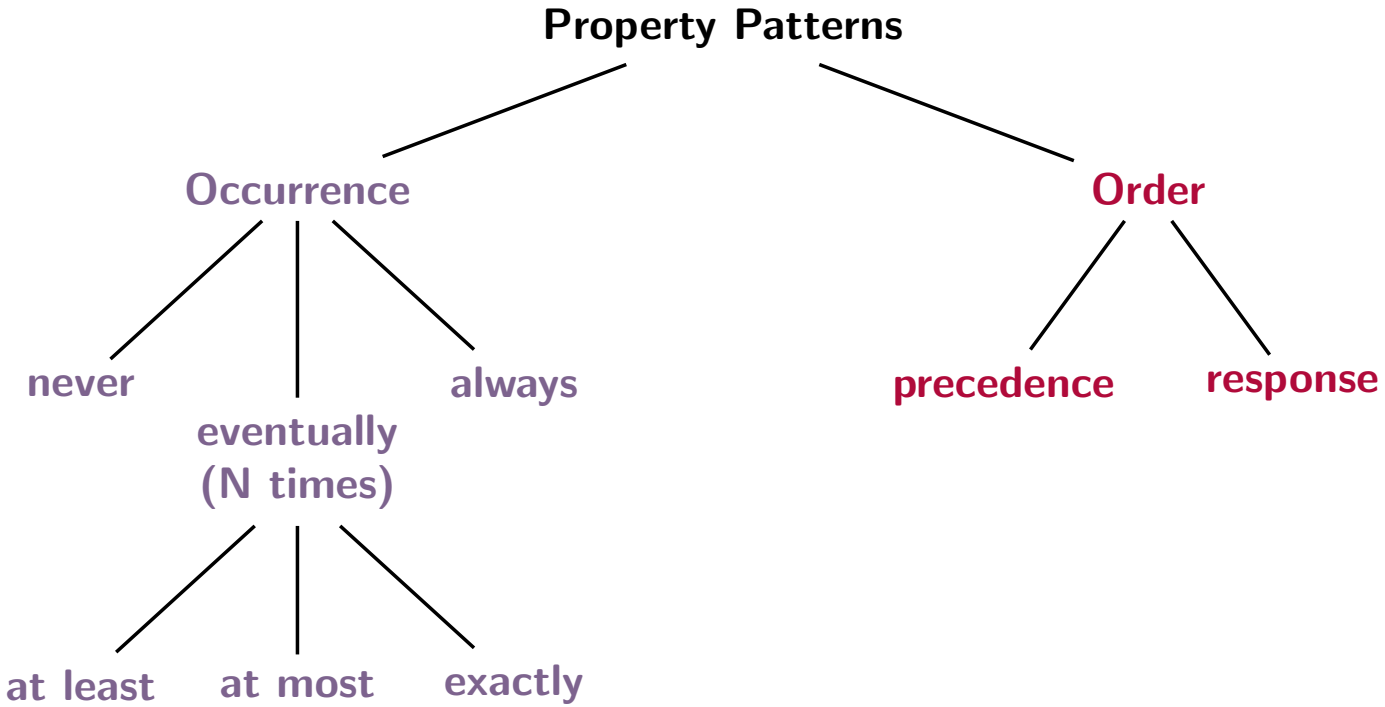

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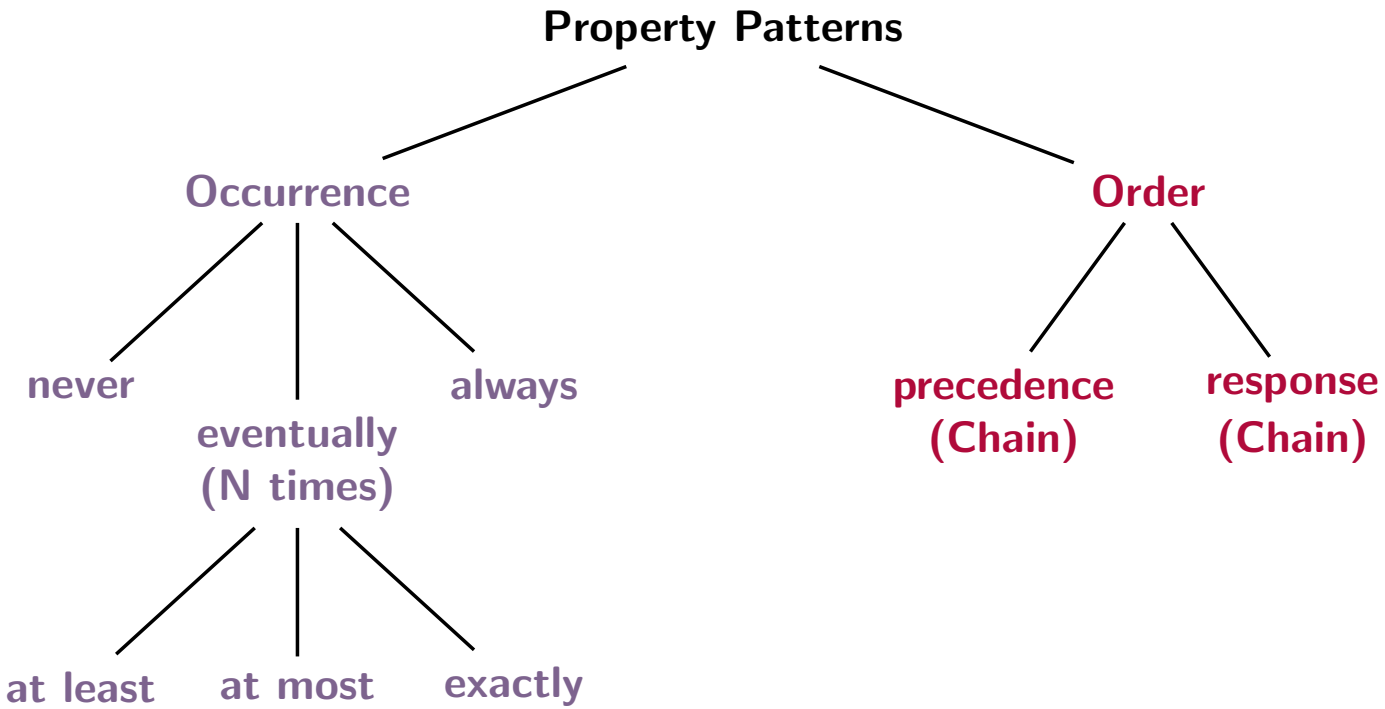

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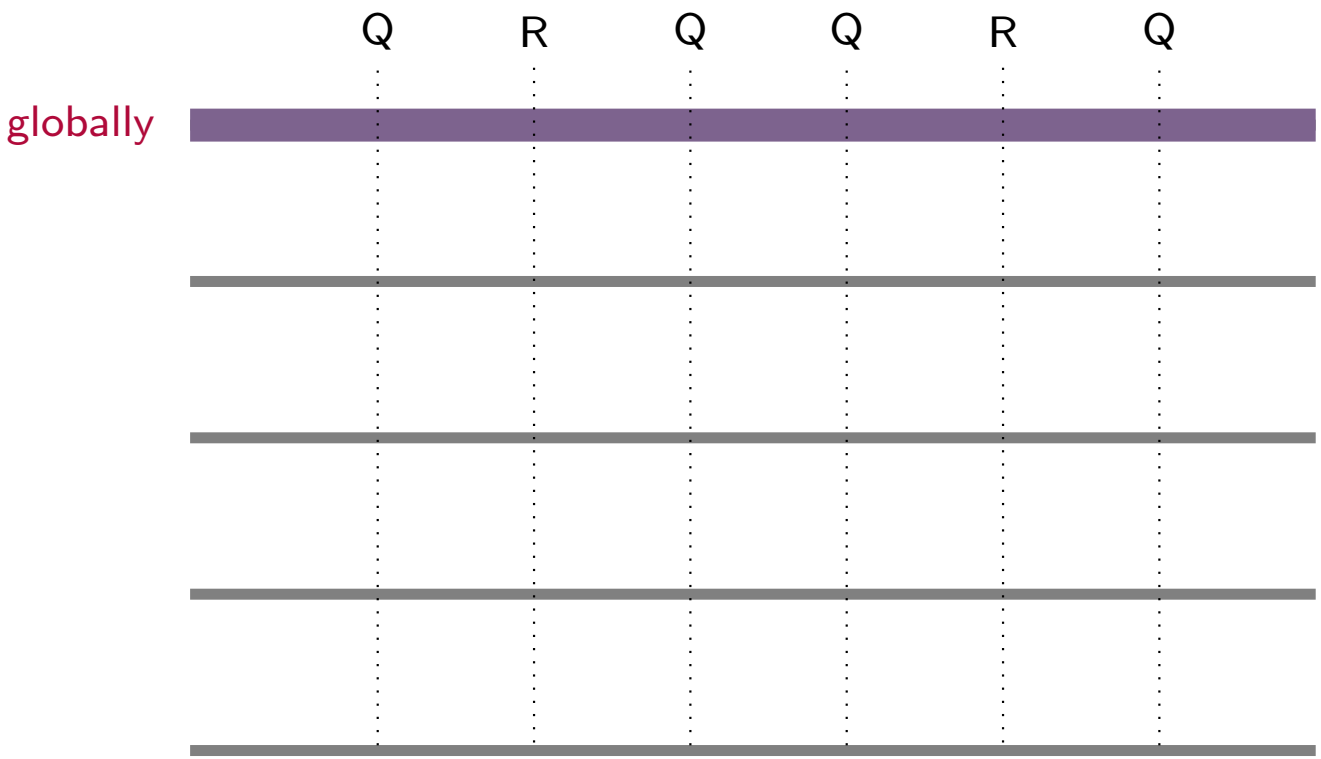

Scopes

The Time Interval(s) over which a pattern holds

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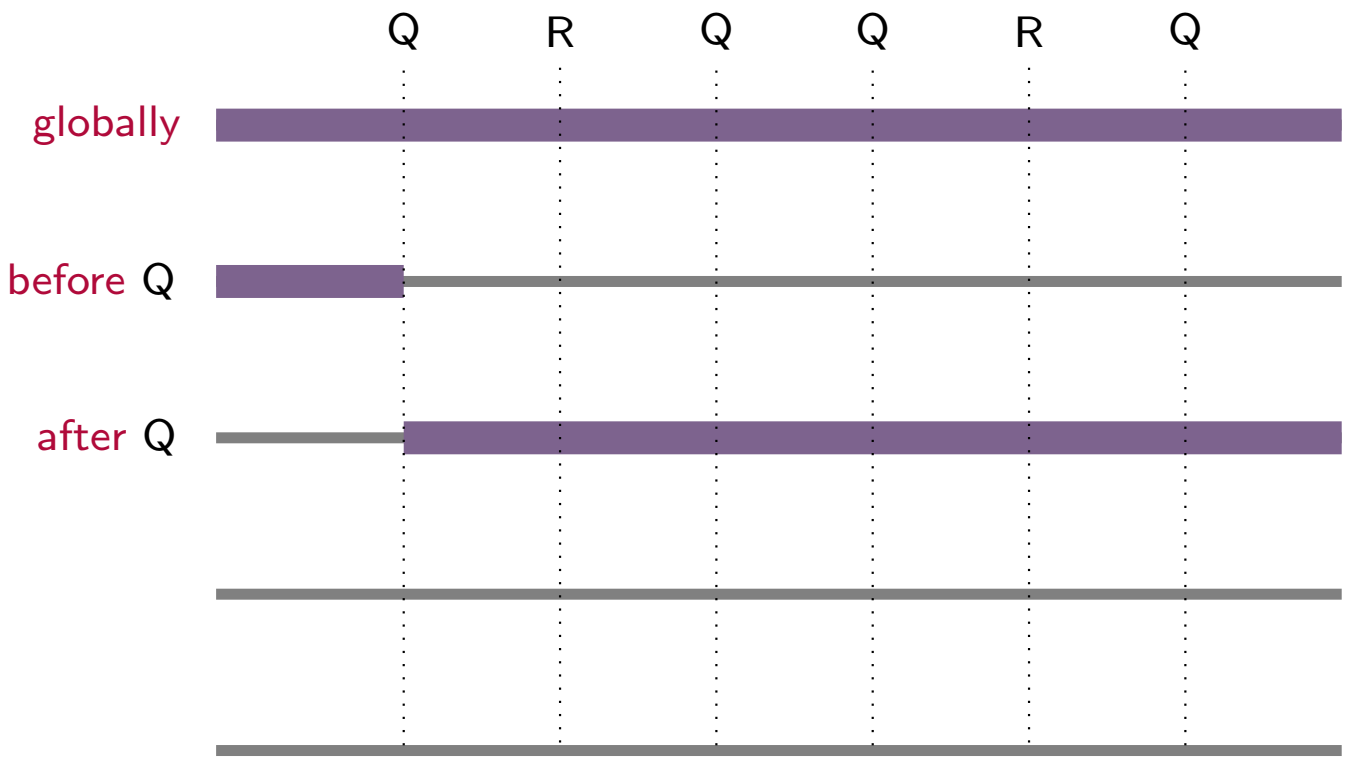

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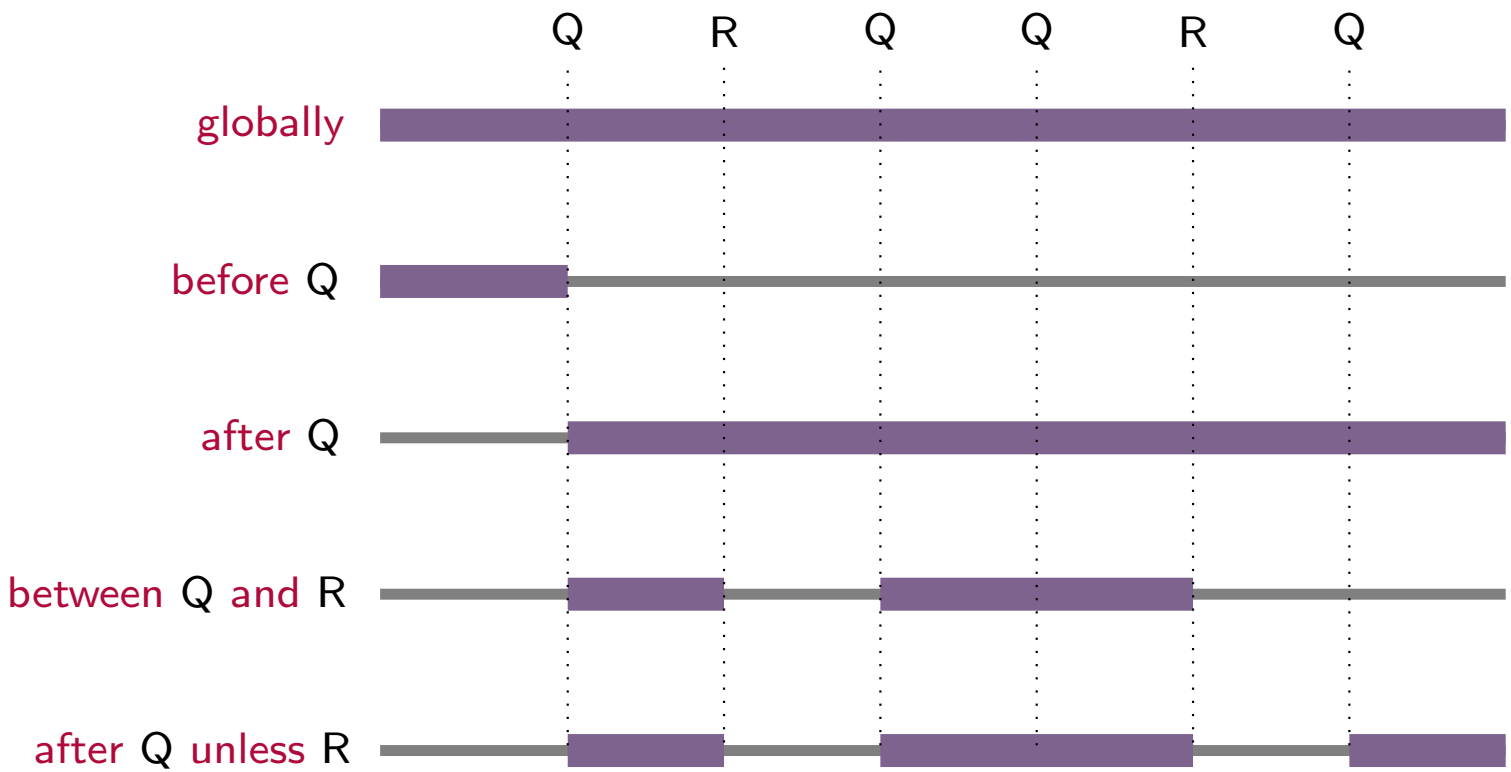

Scopes

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Dwyer et al.'s Spec Patterns

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while a user is connected (between Login and Logout),
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- easy to use
- over $\simeq$ 600 specs : 92% of coverage
  - `http://patterns.projects.cis.ksu.edu/`
- adopted by many works : Bandera, JTPL, Propel ...

Dwyer et al.'s Translational Semantics

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- Dwyer et al. provide formal semantics by translation :
  - $Pattern \times Scope \longrightarrow LTL$

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- Example : $\{Response : P' \text{ responds to } P\} \times Scope \longrightarrow LTL$

Scope s	LTL
globally	$\Box(P \Rightarrow \Diamond P')$
before Q	$\Diamond Q \Rightarrow (P \Rightarrow (\neg Q \ \mathsf{U} \ (P' \wedge \neg Q))) \ \mathsf{U} \ Q$
after Q	$\Box(Q \Rightarrow \Box(P \Rightarrow \Diamond P'))$
between Q `and` R	$\Box((Q \wedge \neg R \wedge \Diamond R) \Rightarrow (P \Rightarrow (\neg R \ \mathsf{U} \ (P' \wedge \neg R))) \ \mathsf{U} \ R)$
after Q `unless` R	$\Box(Q \wedge \neg R \Rightarrow ((P \Rightarrow (\neg R \ \mathsf{U} \ (P' \wedge \neg R))) \ \mathsf{W} \ R)$

Dwyer et al.'s Translational Semantics

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- Dwyer et al. provide formal semantics by translation :
  - $Pattern \times Scope \longrightarrow LTL, CTL, \mu\text{-}calculus, QRE \ldots$
  - Library : `http://patterns.projects.cis.ksu.edu/`

- Example : $\{Response : P' \text{ responds to } P\} \times Scope \longrightarrow LTL$

Scope s	LTL
globally	$\Box(P \Rightarrow \Diamond P')$
before Q	$\Diamond Q \Rightarrow (P \Rightarrow (\neg Q \text{ U } (P' \wedge \neg Q))) \text{ U } Q$
after Q	$\Box(Q \Rightarrow \Box(P \Rightarrow \Diamond P'))$
between Q and R	$\Box((Q \wedge \neg R \wedge \Diamond R) \Rightarrow (P \Rightarrow (\neg R \text{ U } (P' \wedge \neg R))) \text{ U } R)$
after Q unless R	$\Box(Q \wedge \neg R \Rightarrow ((P \Rightarrow (\neg R \text{ U } (P' \wedge \neg R))) \text{ W } R)$

1^{st} Limit : Genericity

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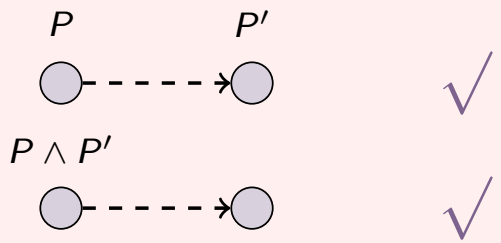
- **without** translating variants :
  - 14 patterns $\times$ 21 scopes $\simeq$ **300** combinations

Patterns Variants

Order Patterns Semantics

- Dwyer et al. consider **non strict** semantics :
  - ex : *P′ responds to P*



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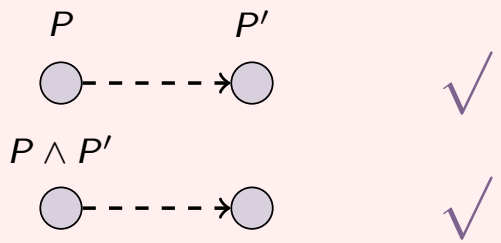
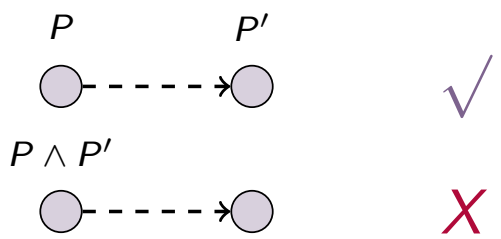


✓
✓

1. Variant : **strict** Order Patterns
   - ex : P' *responds*$_{strictly}$ *to* P



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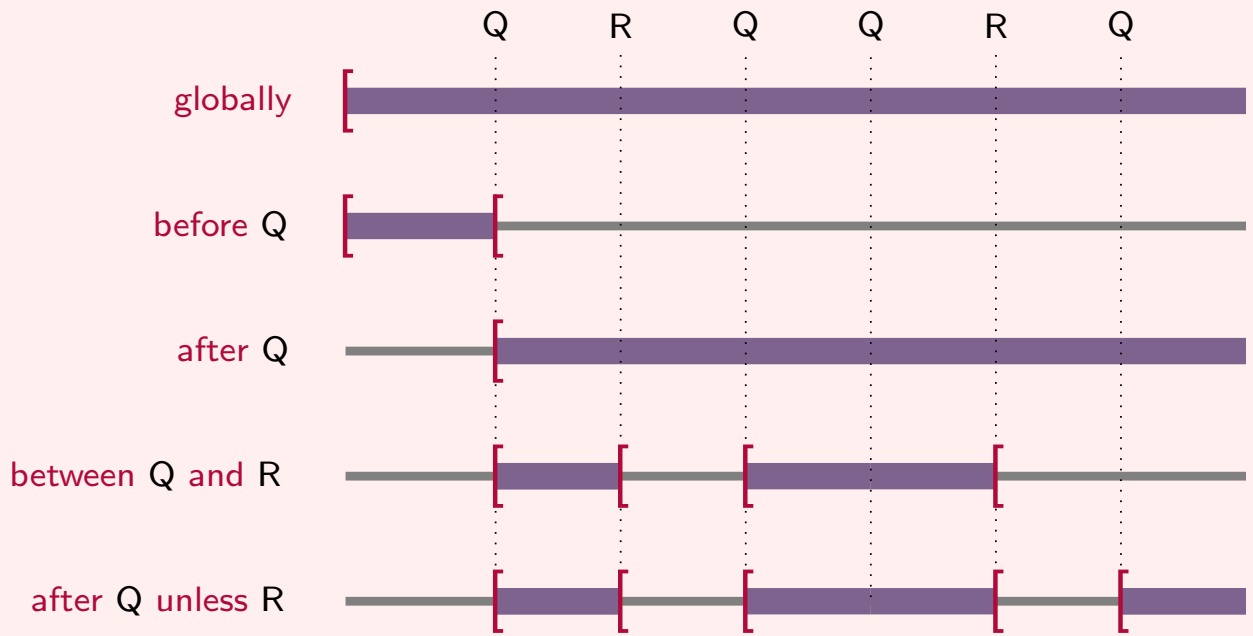
X

Scopes Variants

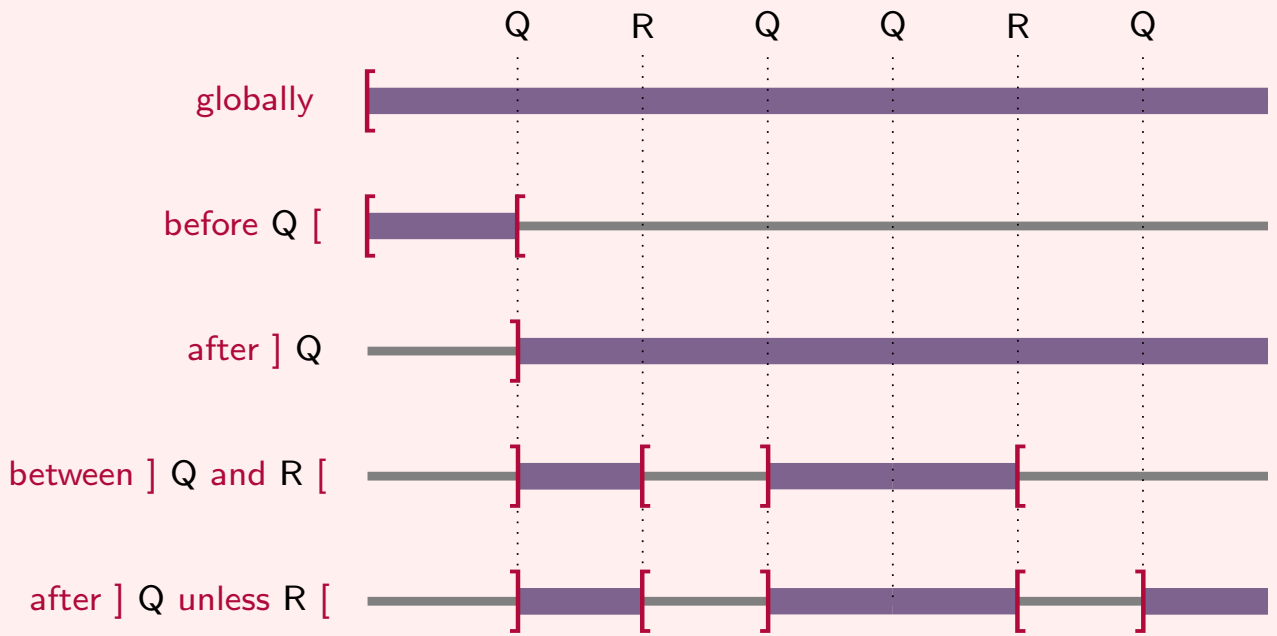
Scopes Semantics



- Dwyer et al. consider right-open intervals
  - [_ , _ [: left-closed and right-open

Scopes Variants

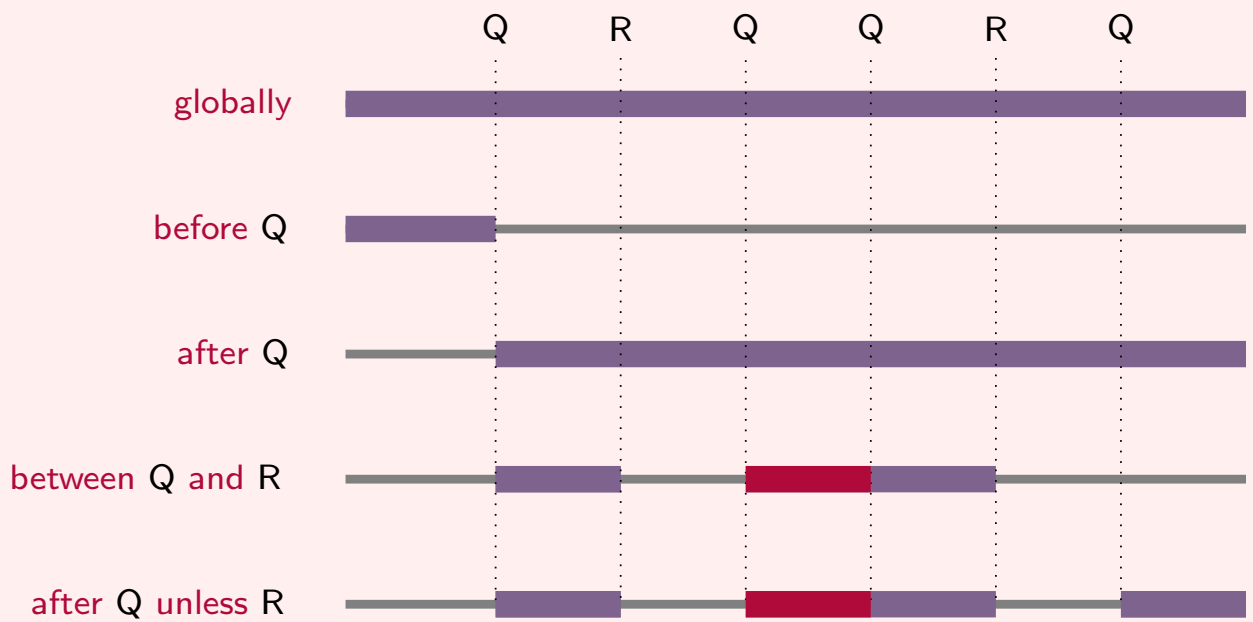
Scopes Semantics



3. Variant : other intervals [_ , _],] _ , _ [et] _ , _]
   - ex : open intervals] _ , _ [

Scopes Variants

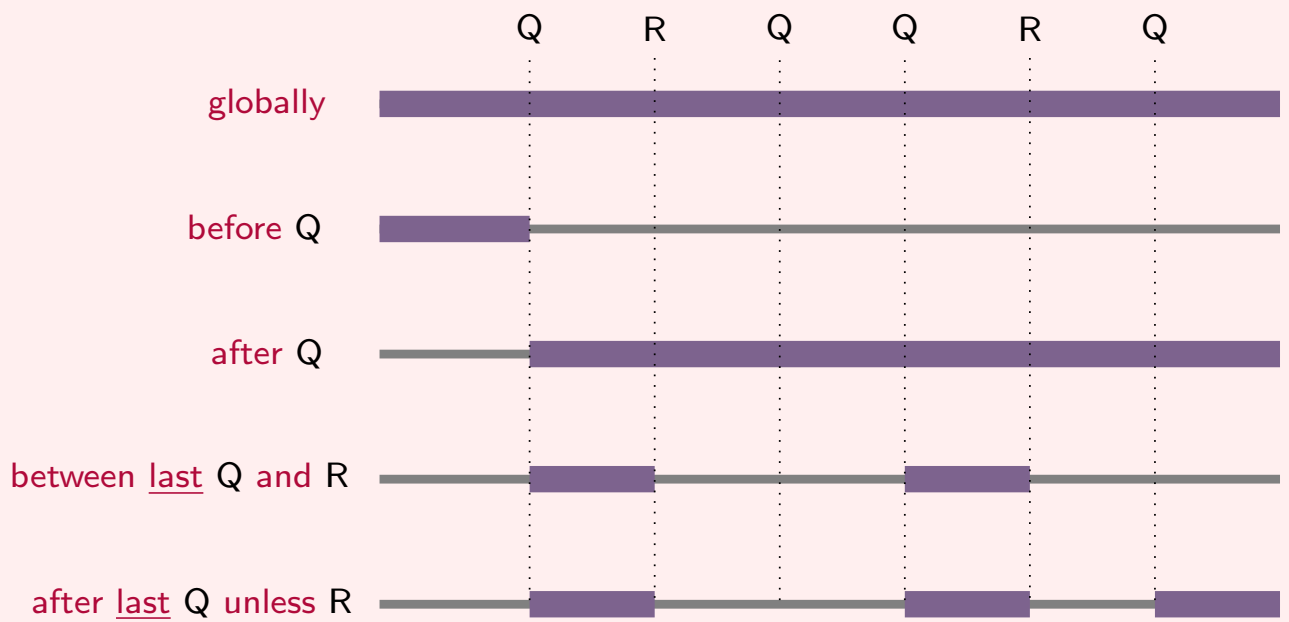
Scopes Semantics



- Dwyer et al. consider the first occurrence

Scopes Variants

Scopes Semantics



4. Variant : select the last occurrence

3^{rd} Limit : Faithfulness

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  - Peer-review among the Dwyer et al. project members
  - Model-checking the LTL formulas against some (un)satisfying scenarios (w.r.t. natural semantics)
- Common issue between natural and formal semantics :
  - No way ! to measure faithfulness !

3^{rd} Limit : Faithfulness

- Dwyer et al. build the Property Specification Language over a clear separation between Patterns and Scopes
  - A Pattern (resp. Scope) has a unique natural semantics regardless the Scope (resp. Pattern) with which it is combined

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  - ex : a Pattern is translated 5 times and may have up to 5 different interpretations
- Homogeneity is part of Faithfulness
  - a part that we will be able to measure !

Plan

Principle [Taha et al., 2015]

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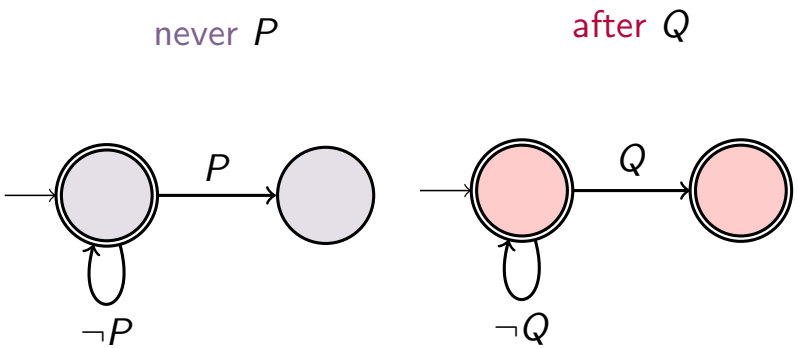

1. Pattern and Scope are Büchi automata

Principle [Taha et al., 2015]

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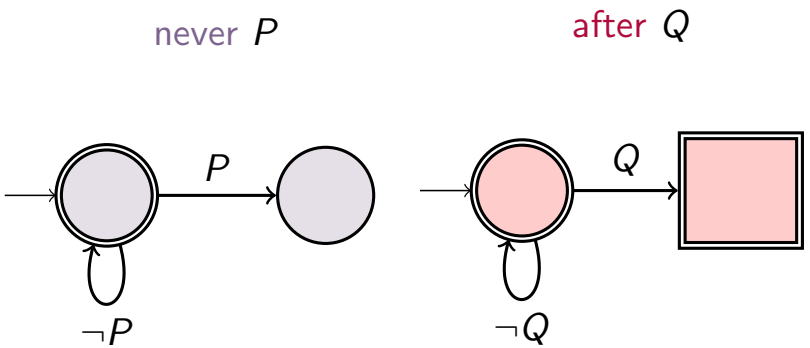

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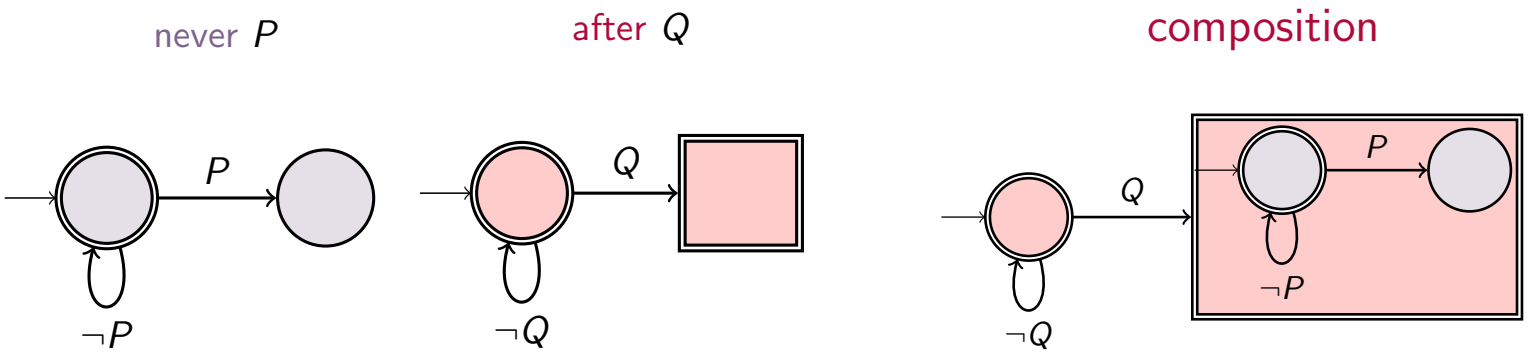

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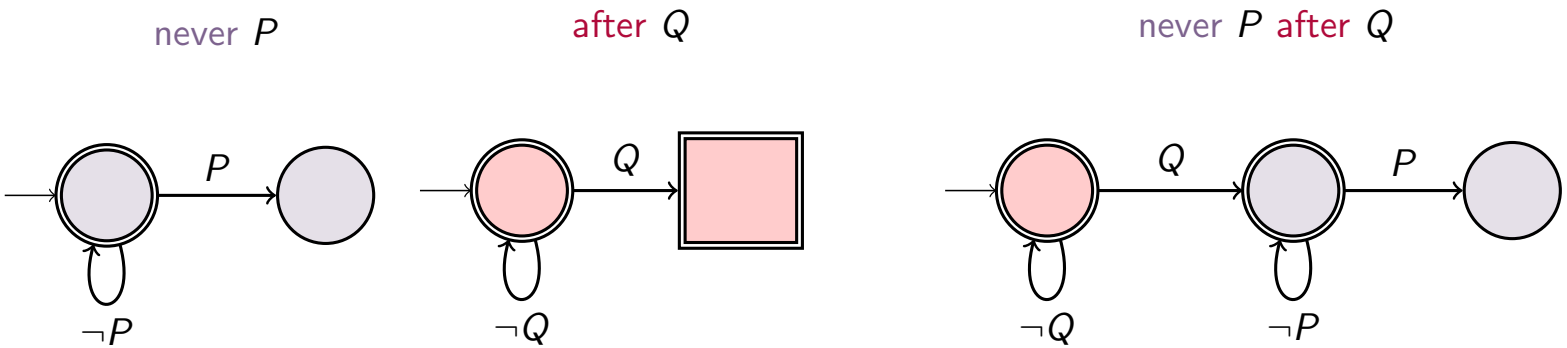

1. Pattern and Scope are Büchi automata
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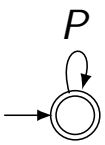
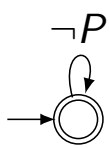
4. We obtain the automaton of the property : [Pattern/Scope]
   - by applying composition rules

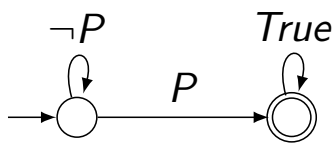
Patterns Automata



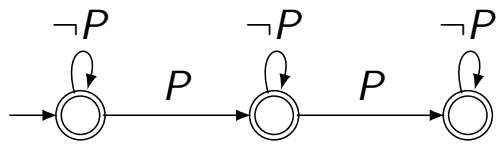
always P



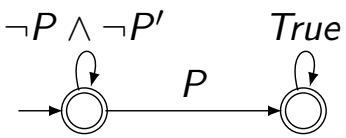
never P



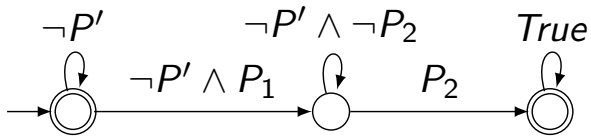
eventually P



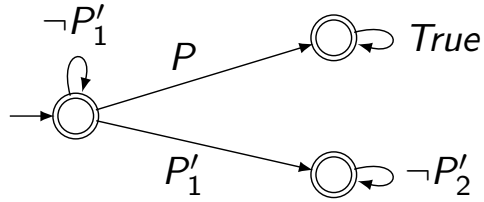
eventually P at most 2 times



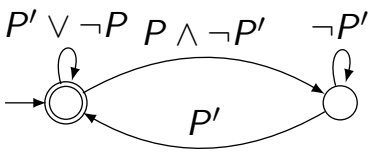
P precedes P′



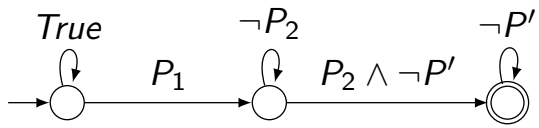
(P_1, P_2) precedes P′



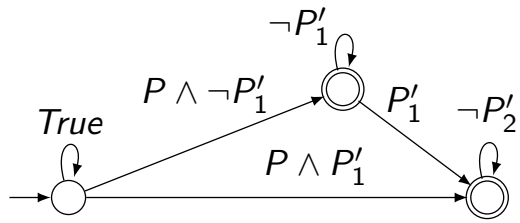
P precedes (P'_1, P'_2)



P′ responds to P

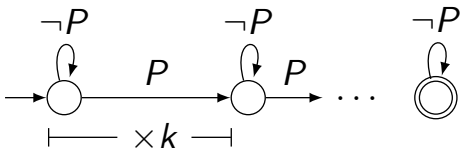


¬[P′ responds to (P_1, P_2)]

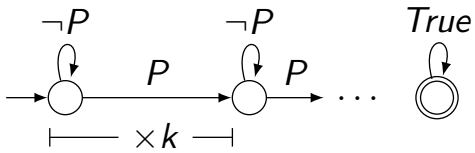


¬[(P'_1, P'_2) responds to P]

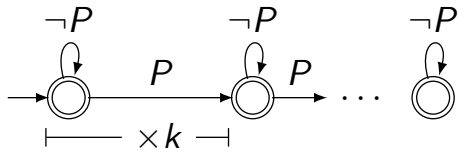
Patterns Automata + generic



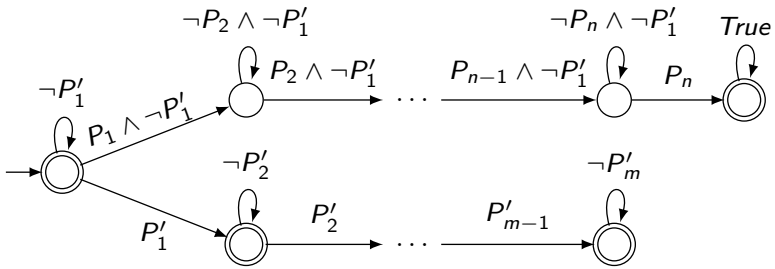
eventually P k times



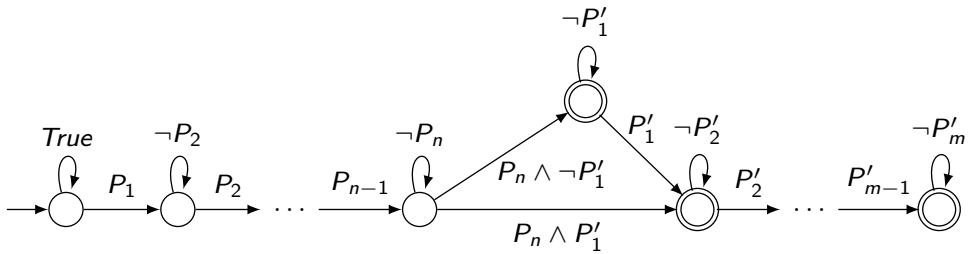
eventually P at least k times



eventually P at most k times

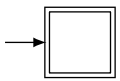


$(P_1, \ldots, P_n)$ precedes $(P'_1, \ldots, P'_m)$

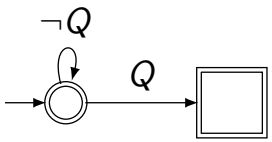


$\neg[(P'_1, \ldots, P'_m)$ responds to $(P_1, \ldots, P_n)]$

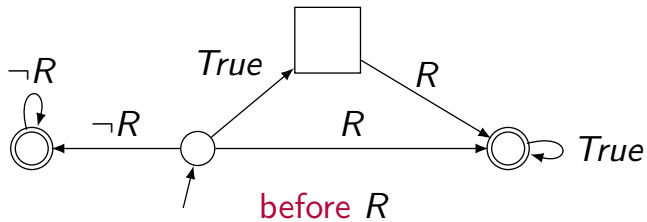
Scopes Automata



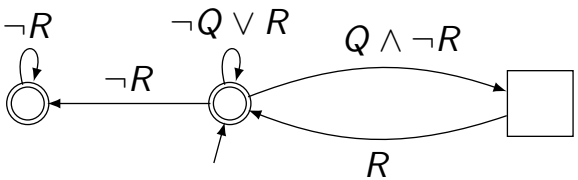
globally



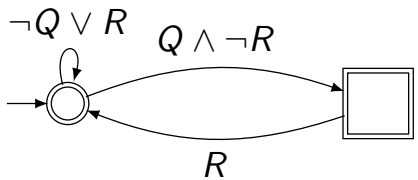
after Q



before R

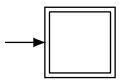


between Q and R

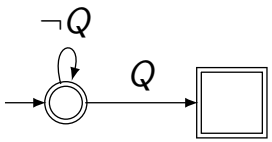


after Q unless R

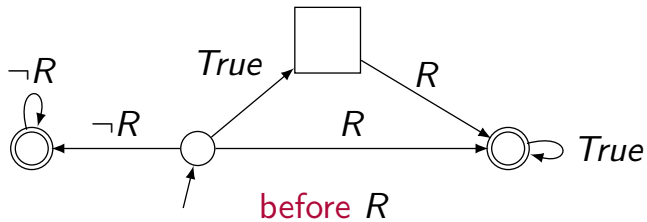
Scopes Automata + variants



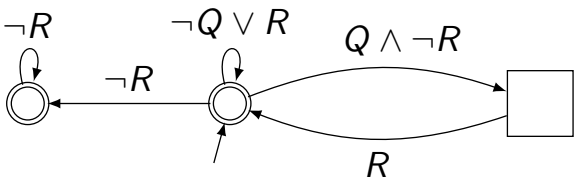
globally



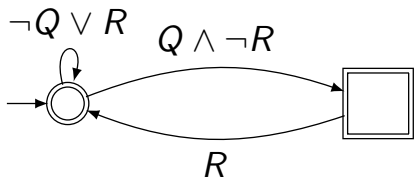
after Q



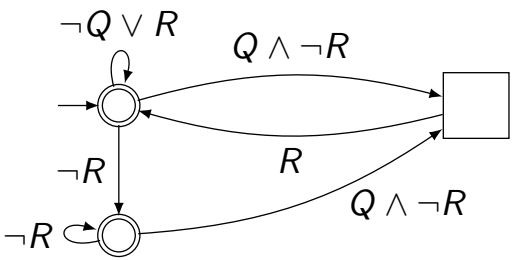
before R



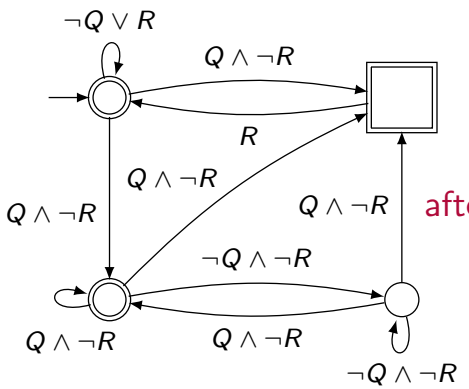
between Q and R



after Q unless R



between last Q and R



after last Q unless R

Composition Rules : States

$$\begin{cases} \text{pattern} & \stackrel{\text{def}}{=} (Q_p, \quad init_p, F_p, T_p) \quad with \quad init_p \in Q_p, F_p \subseteq Q_p \\ \text{scope} & \stackrel{\text{def}}{=} (Q_s \cup \{cs\}, init_s, F_s, T_s) \quad with \quad init_s \in Q_s \cup \{cs\}, F_s \subseteq Q_s \cup \{cs\} \end{cases}$$

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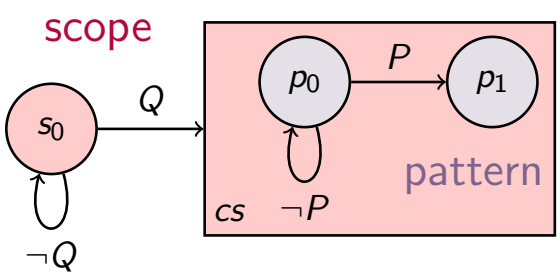
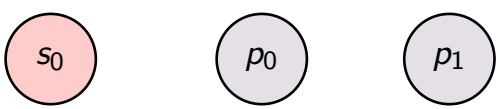
pattern/scope $\stackrel{\text{def}}{=} (Q, init, F, T)$

$Q = Q_p \cup Q_s$





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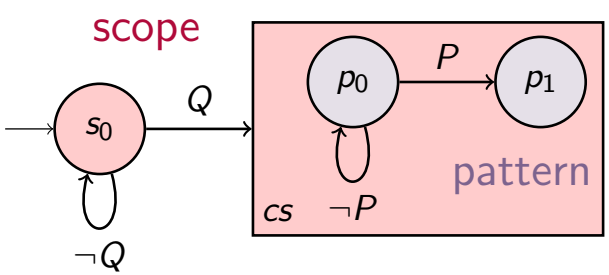

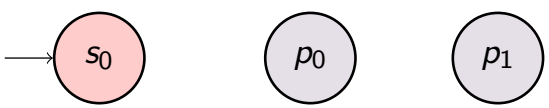

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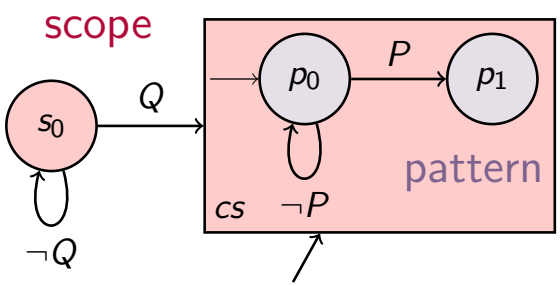

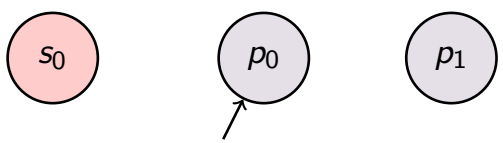

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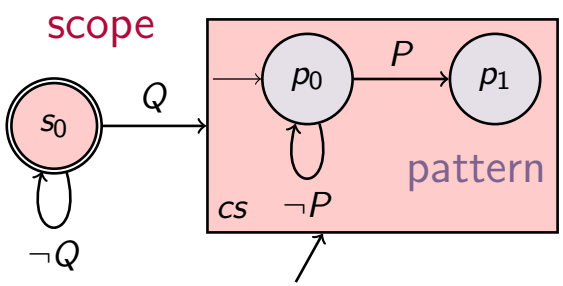

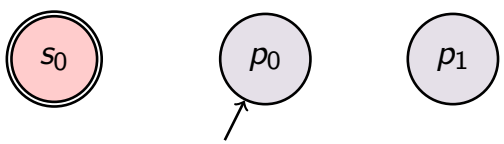

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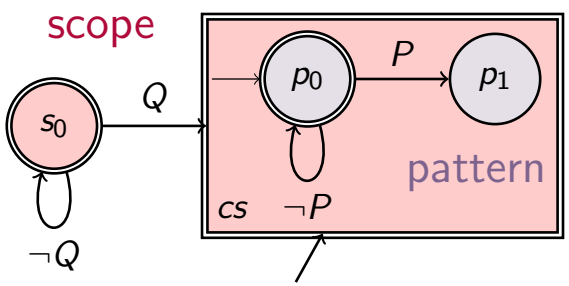
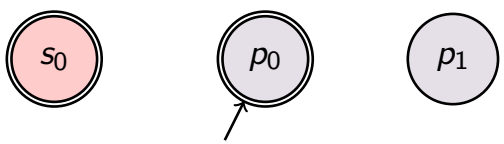
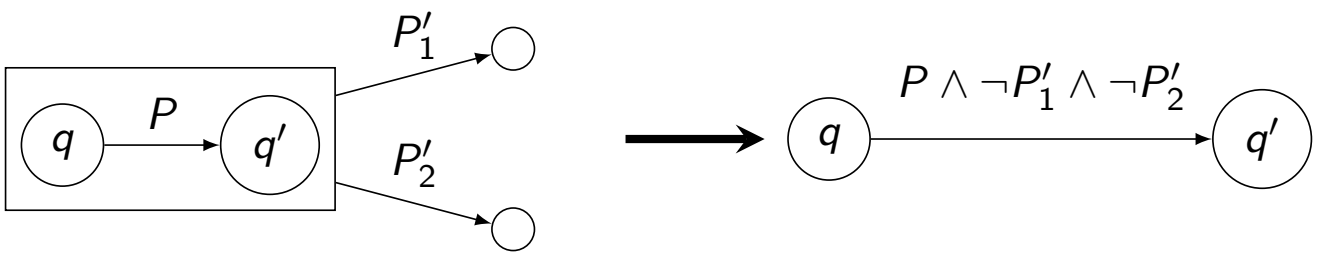




Composition Rules : Transitions (1/3)

1. Transitions from pattern to pattern : restriction

$$\frac{q \xrightarrow{P} q' \in T_p}{q \xrightarrow{P \wedge R} q' \in T} \quad \text{avec} \quad \begin{cases} R & = \bigwedge_{P' \in Out(cs)} \neg P' \\ Out(cs) & = \{P' \mid \exists q'', q'' \in Q_s, cs \xrightarrow{P'} q'' \in T_s\} \end{cases}$$



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$$\frac{q \xrightarrow{P} q' \in T_s,\ q,\ q' \in Q_s}{q \xrightarrow{P} q' \in T}$$

Composition Rules : Transitions (2/3)

3. Transitions from scope to pattern :

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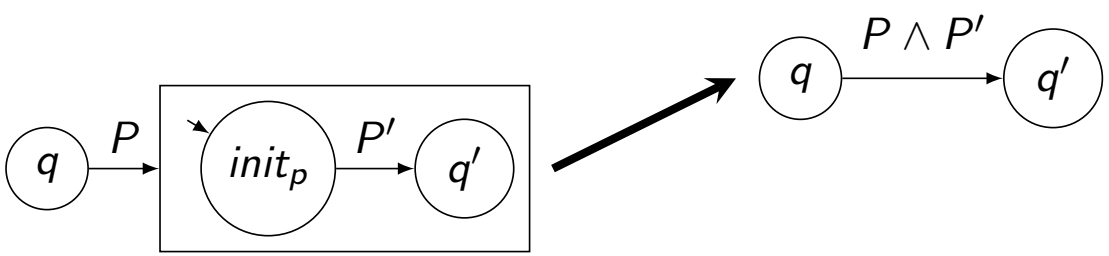

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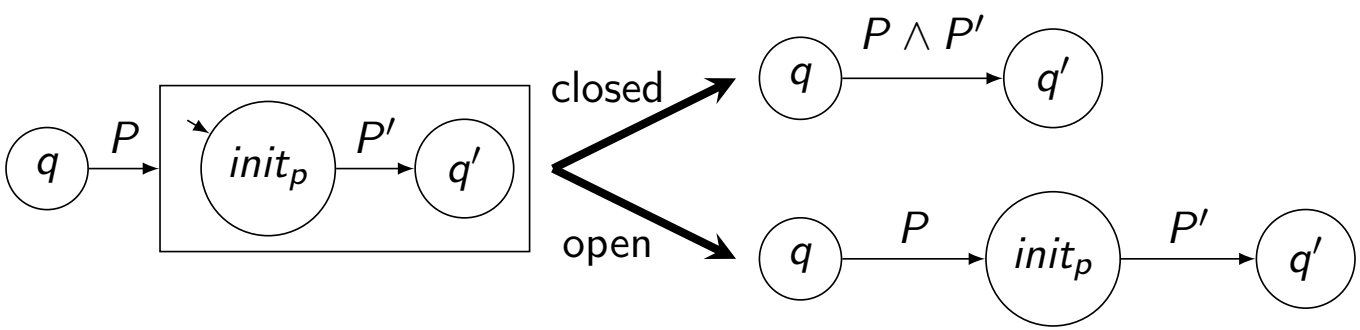

Composition Rules : Transitions (3/3)

4. Transitions from pattern to scope :
   - right-open interval :

$$\frac{cs \xrightarrow{P} q' \in T_s,\ q' \in Q_s,\ q \in F_p}{q \xrightarrow{P} q' \in T}$$

   - right-closed interval (variant) :

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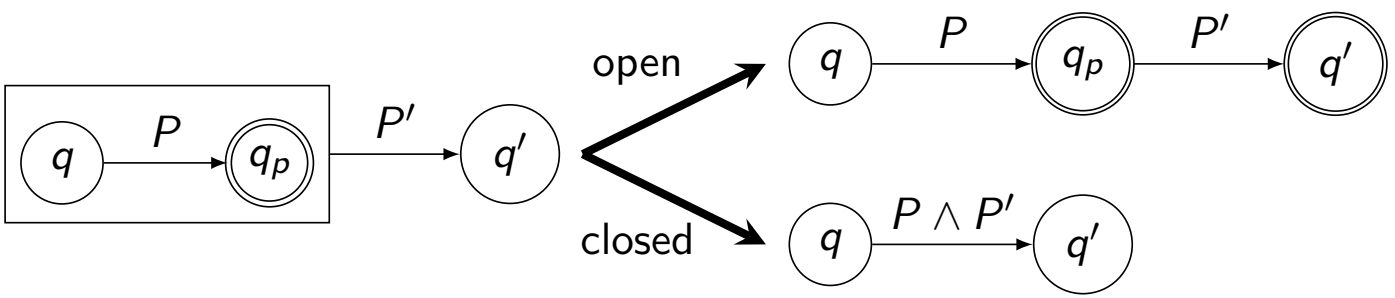

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while a user is connected (between Login and Logout), each user request (Req) must be responded (Resp)

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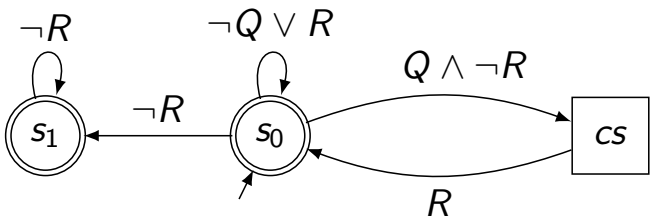

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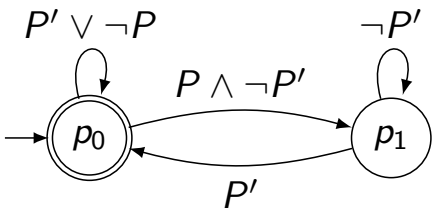

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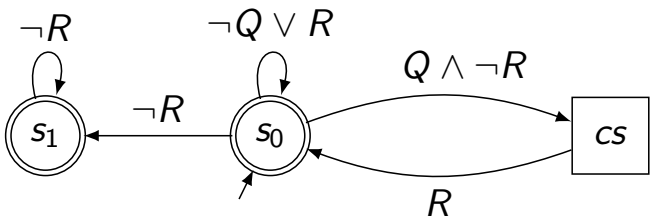

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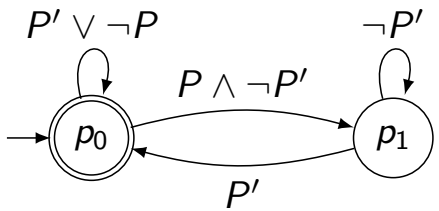

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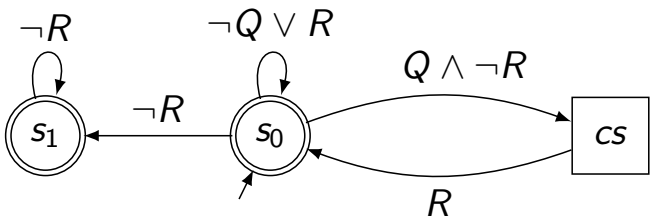

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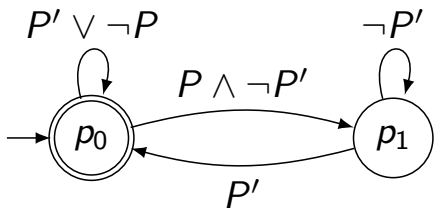

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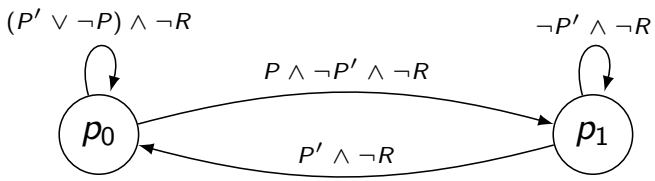

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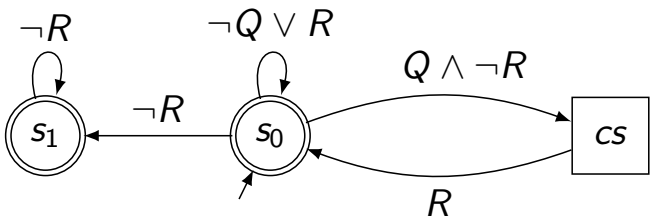

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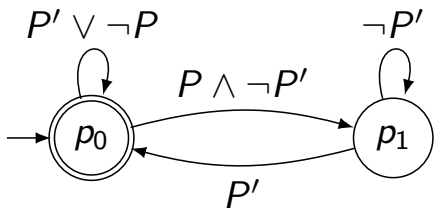

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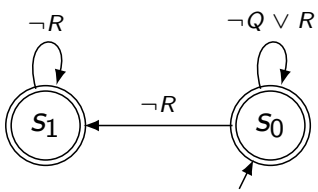

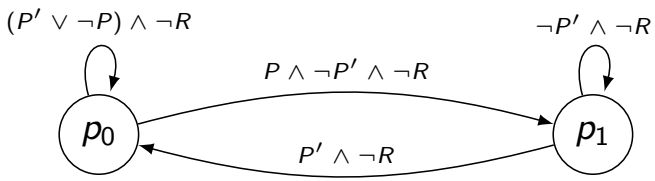

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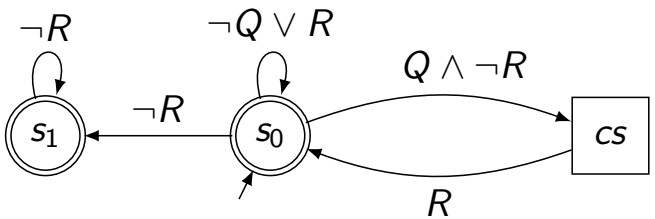

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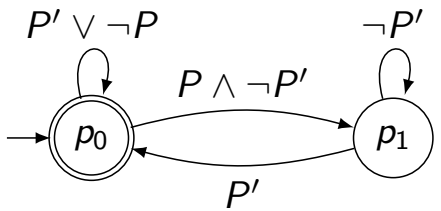

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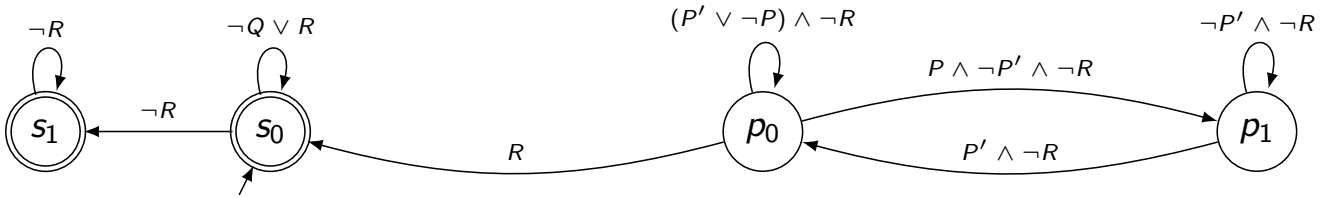

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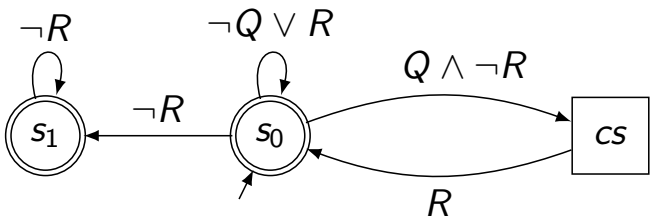

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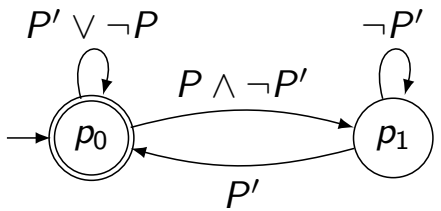

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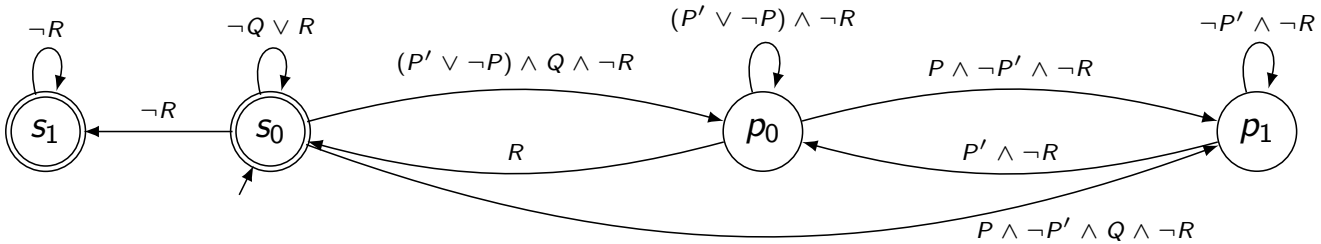

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- Translational Semantics by Dwyer et al. :
  - n patterns $\times$ m scopes $\simeq$ **n** $\times$ **m** combinations to translate
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Composition **2 : 0** **Translation**

- Compositional Semantics brings Genericity and Scalability
- what about Homogeneity ?

Homogeneity

	globally	before R	after Q	between Q and R	after Q unless R
always P	$\Box P$	$\Diamond R \Rightarrow (P \text{ U } R)$	$\Box(Q \Rightarrow \Box P)$	$\Box((Q \wedge \neg R \wedge \Diamond R) \Rightarrow (P \text{ U } R))$	$\Box((Q \wedge \neg R) \Rightarrow (P \text{ W } R))$
never P	$\Box\neg P$	LTL	LTL	LTL	LTL
eventually P	$\Diamond P$	LTL	LTL	LTL	LTL
eventually P at most 2	$(\neg P \text{ W } (P \text{ W } (\neg P \text{ W } (P \text{ W } \Box\neg P))))$	LTL	LTL	LTL	LTL
P precedes P′	$\neg P' \text{ W } P$	LTL	LTL	LTL	LTL
(P_1, P_2) precedes P′	$\Diamond P' \Rightarrow (\neg P' \text{ U } (P_1 \wedge \neg P' \wedge \bigcirc(\neg P' \text{ U } P_2)))$	LTL	LTL	LTL	LTL
P precedes (P_1', P_2')	$(\Diamond(P_1' \wedge \bigcirc\Diamond P_2')) \Rightarrow ((\neg P_1') \text{ U } P))$	LTL	LTL	LTL	LTL
P′ responds to P	$\Box(P \Rightarrow \Diamond P')$	LTL	LTL	LTL	LTL
(P_1', P_2') responds to P	$\Box(P \Rightarrow \Diamond(P_1' \wedge \bigcirc\Diamond P_2'))$	LTL	LTL	LTL	LTL
P′ responds to (P_1, P_2)	$\Box(P_1 \wedge \bigcirc\Diamond P_2 \Rightarrow \bigcirc(\Diamond(P_2 \wedge \Diamond P')))$	LTL	LTL	LTL	LTL

Approach :

1. Collect the 50 LTL formulas given by Dwyer et al.

Homogeneity

	globally
always P	$\Box P$
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eventually P	$\Diamond P$
eventually P at most 2	$(\neg P \text{ W } (P \text{ W } (\neg P \text{ W } (P \text{ W } \Box\neg P))))$
P precedes P′	$\neg P' \text{ W } P$
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P precedes (P'_1, P'_2)	$(\Diamond(P'_1 \wedge \bigcirc\Diamond P'_2)) \Rightarrow ((\neg P'_1) \text{ U } P))$
P′ responds to P	$\Box(P \Rightarrow \Diamond P')$
(P'_1, P'_2) responds to P	$\Box(P \Rightarrow \Diamond(P'_1 \wedge \bigcirc\Diamond P'_2))$
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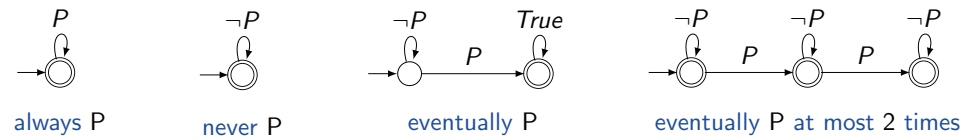

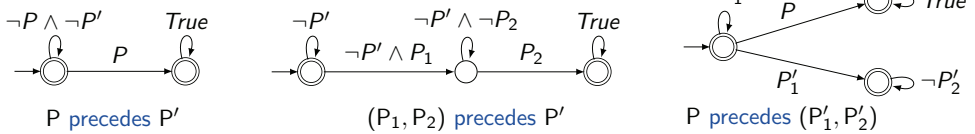

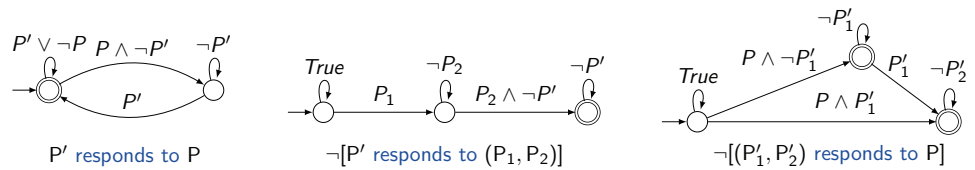

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2. Generate the Patterns automata (LTL2BA)

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	globally	before R	after Q	between Q and R	after Q unless R
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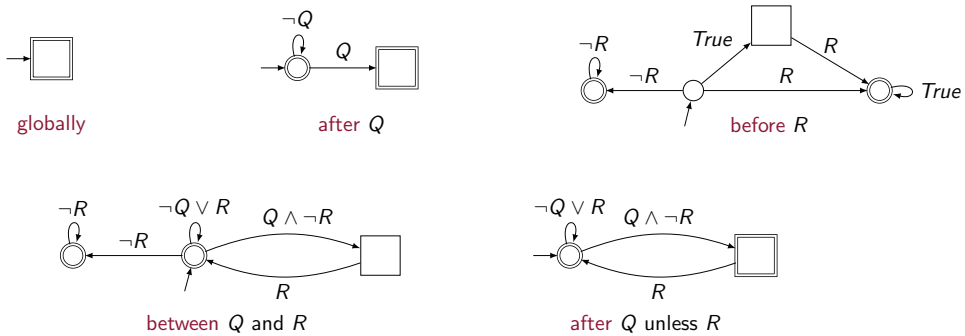

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1. Collect the 50 LTL formulas given by Dwyer et al.
2. Generate the Patterns automata (LTL2BA)
3. Generate the Scopes automata (LTL2BA)

Homogeneity

	globally	before R	after Q	between Q and R	after Q unless R
always P	$\Box P$	$\Diamond R \Rightarrow (P \text{ U } R)$	$\Box(Q \Rightarrow \Box P)$	$\Box((Q \wedge \neg R \wedge \Diamond R) \Rightarrow (P \text{ U } R))$	$\Box((Q \wedge \neg R) \Rightarrow (P \text{ W } R))$
never P	$\Box\neg P$	Composition	Composition	Composition	Composition
eventually P	$\Diamond P$	Composition	Composition	Composition	Composition
eventually P at most 2	$(\neg P \text{ W } (P \text{ W } (\neg P \text{ W } (P \text{ W } \Box\neg P))))$	Composition	Composition	Composition	Composition
P precedes P′	$\neg P' \text{ W } P$	Composition	Composition	Composition	Composition
(P_1, P_2) precedes P′	$\Diamond P' \Rightarrow (\neg P' \text{ U } (P_1 \wedge \neg P' \wedge \bigcirc(\neg P' \text{ U } P_2)))$	Composition	Composition	Composition	Composition
P precedes (P'_1, P'_2)	$(\Diamond(P'_1 \wedge \bigcirc\Diamond P'_2)) \Rightarrow ((\neg P'_1) \text{ U } P))$	Composition	Composition	Composition	Composition
P′responds to P	$\Box(P \Rightarrow \Diamond P')$	Composition	Composition	Composition	Composition
(P'_1, P'_2) responds to P	$\Box(P \Rightarrow \Diamond(P'_1 \wedge \bigcirc\Diamond P'_2))$	Composition	Composition	Composition	Composition
P′ responds to (P_1, P_2)	$\Box(P_1 \wedge \bigcirc\Diamond P_2 \Rightarrow \bigcirc(\Diamond(P_2 \wedge \Diamond P')))$	Composition	Composition	Composition	Composition

Approach :

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never P	$\Box\neg P$	$\equiv$	$\equiv$	$\equiv$	$\equiv$
eventually P	$\Diamond P$	$\subset$ (1)	$\equiv$	$\subset$ (2)	$\subset$ (2)
eventually P at most 2	$(\neg P \text{ W } (P \text{ W } (\neg P \text{ W } (P \text{ W } \Box\neg P))))$	$\equiv$	$\equiv$	$\equiv$	$\equiv$
P precedes P′	$\neg P' \text{ W } P$	$\equiv$	$\supset$ (3)	$\subset$ (2)	$\subset$ (2)
(P_1, P_2) precedes P′	$\Diamond P' \Rightarrow (\neg P' \text{ U } (P_1 \wedge \neg P' \wedge \bigcirc(\neg P' \text{ U } P_2)))$	$\equiv$	$\equiv$	$\subset$ (2)	$\subset$ (2)
P precedes (P'_1, P'_2)	$(\Diamond(P'_1 \wedge \bigcirc\Diamond P'_2)) \Rightarrow ((\neg P'_1) \text{ U } P))$	$\equiv$	$\equiv$	$\subset$ (2)	$\subset$ (2)
P′ responds to P	$\Box(P \Rightarrow \Diamond P')$	$\equiv$	$\equiv$	$\equiv$	$\equiv$
(P'_1, P'_2) responds to P	$\Box(P \Rightarrow \Diamond(P'_1 \wedge \bigcirc\Diamond P'_2))$	$\supset$ (4)	$\equiv$	$\supset$ (4)	$\supset$ (4)
P′ responds to (P_1, P_2)	$\Box(P_1 \wedge \bigcirc\Diamond P_2 \Rightarrow \bigcirc(\Diamond(P_2 \wedge \Diamond P')))$	$\neq$ (5)	$\equiv$	$\neq$ (5)	$\neq$ (5)

Approach :

1. Collect the 50 LTL formulas given by Dwyer et al.
2. Generate the Patterns automata (LTL2BA)
3. Generate the Scopes automata (LTL2BA)
4. Apply the composition to generate the automata of remaining combinations
5. Compare ! (Translation $\equiv, \subset, \supset$ Composition)

Homogeneity : mismatching cases

Mismatching cases	**Dwyer et al.'s Formula**		**Composition Formula**
(1) eventually P before R	$\neg R \ \mathsf{W} \ (P \wedge \neg R)$	$\subset$	
(2) P precedes P′ / P eventually between Q and R / after Q unless R	$\Box((Q \wedge \neg R) \Rightarrow \dots)$	$\subset$	
(3) P precedes P′ after Q	$\Box\neg Q \vee \Diamond(Q \wedge (\neg P' \ \mathsf{W} \ P))$	$\supset$	
(4) $(\mathrm{P}'_1, \mathrm{P}'_2)$ responds to P before R/ . . .	$\dots(P \Rightarrow (\neg R \ \mathsf{U} \ (P'_1 \wedge \neg R \wedge \bigcirc(\neg R \ \mathsf{U} \ P'_2)))) \dots$	$\supset$	
(5) P′ responds to $(\mathrm{P}_1, \mathrm{P}_2)$ before R/ . . .	$\dots(P_1 \wedge \bigcirc(\neg R \ \mathsf{U} \ P_2) \Rightarrow \bigcirc(\neg R \ \mathsf{U} \ (P_2 \wedge \Diamond P'))) \dots$	$\neq$	

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(1) eventually P before R	$\neg R \ \mathsf{W}\ (P \wedge \neg R)$	$\subset$	$\underline{R \vee} \ \neg R \ \mathsf{W}\ (P \wedge \neg R)$
(2) P precedes P′ / P eventually between Q and R / after Q unless R	$\Box((Q \wedge \neg R) \Rightarrow \dots)$	$\subset$	
(3) P precedes P′ after Q	$\Box\neg Q \vee \Diamond(Q \wedge (\neg P' \ \mathsf{W}\ P))$	$\supset$	
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(5) P′ responds to (P_1, P_2) before R/ . . .	$\dots(P_1 \wedge \bigcirc(\neg R \ \mathsf{U}\ P_2) \Rightarrow \bigcirc(\neg R \ \mathsf{U}\ (P_2 \wedge \Diamond P'))) \dots$	$\neq$	

1. if R holds immediately (right-open interval) !

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(1) eventually P before R	$\neg R \ \mathsf{W} \ (P \wedge \neg R)$	$\subset$	$R \vee \neg R \ \mathsf{W} \ (P \wedge \neg R)$
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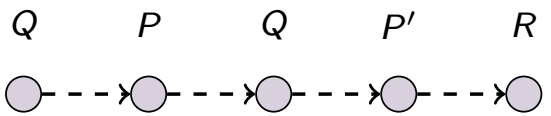
1. if R holds immediately (right-open interval) !
2. Dwyer et al. consider all occurrences of Q (not the 1^{st}) : stronger constraint



Translation ✗ Composition ✓

Homogeneity : mismatching cases

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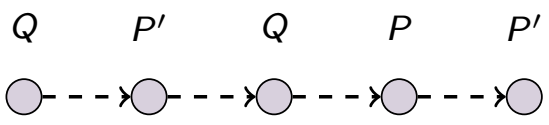

Translation $\surd$ Composition X

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Composition 3 : 0 Translation

- Compositional Semantics reveals inconsistencies within Translational Semantics

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- Compositional Semantics reveals inconsistencies within Translational Semantics

Composition 4 : 0 Translation

- Composition is linear where $LTL \rightarrow BA$ is PSPACE-complete !

Furthermore

- Dwyer et al. combine 1 pattern and 1 scope

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Extension

- 1 pattern and successive applications of k scopes !

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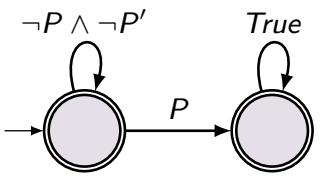

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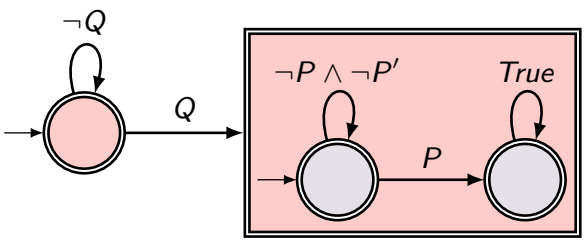
- ex : P precedes P' after Q



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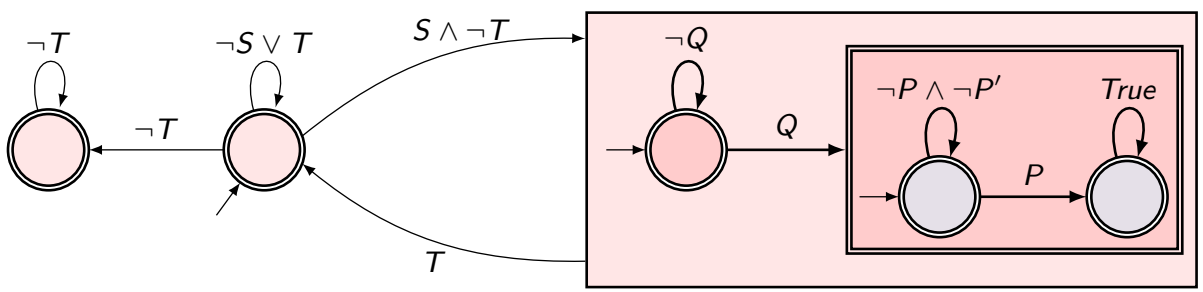
- ex : P precedes P' after Q, between S and T



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Extension

- 1 pattern and successive applications of k scopes !
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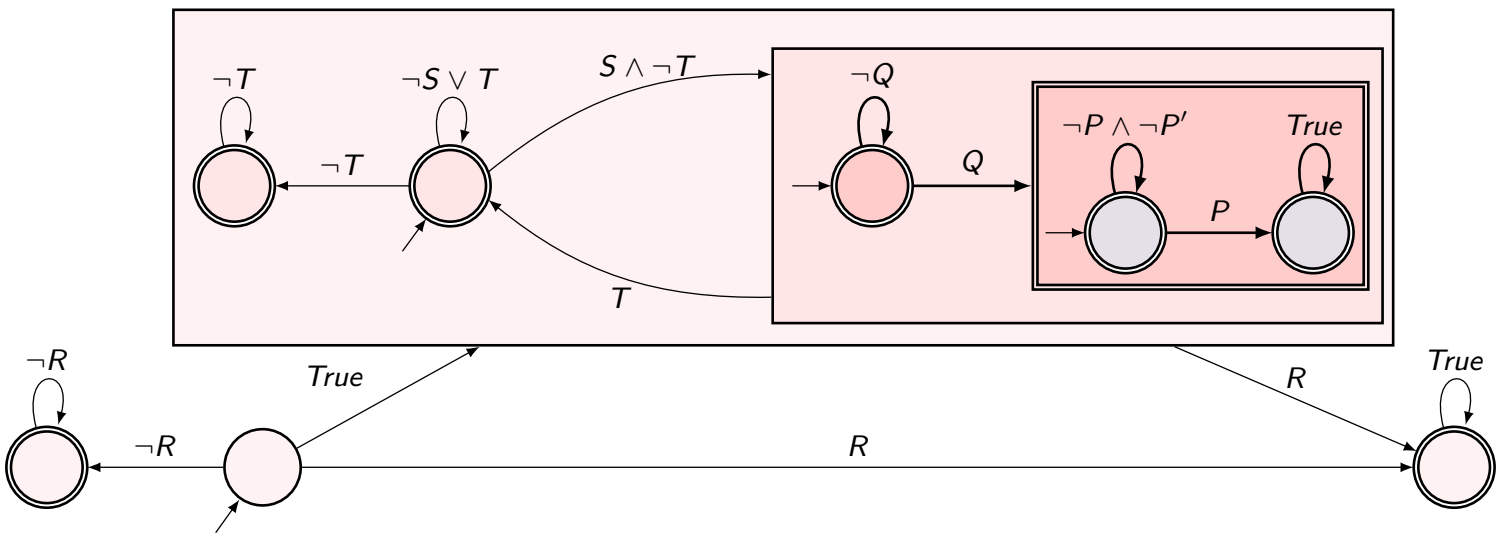
- ex : P precedes P' after Q, between S and T, before R ...



Plan

Test Intention

- Given a test purpose automaton, coverage criteria may be :
  - all states
  - all transitions
  - all k-paths
  - ...

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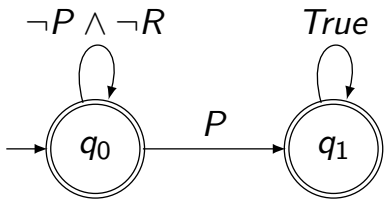
Transformation rules

There is a common practice to transform a test purpose automaton to an **equivalent** test intention automaton

Example : P precedes R



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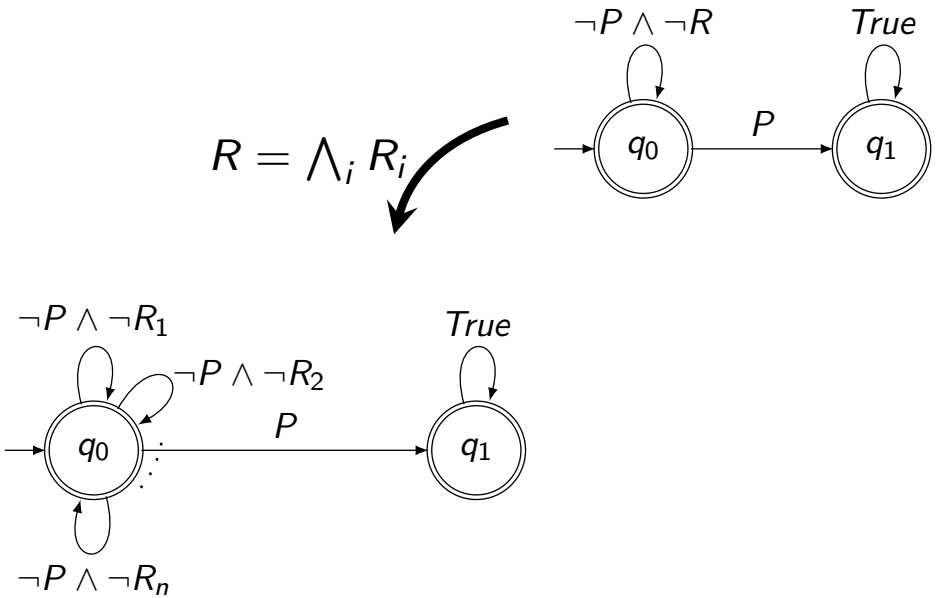

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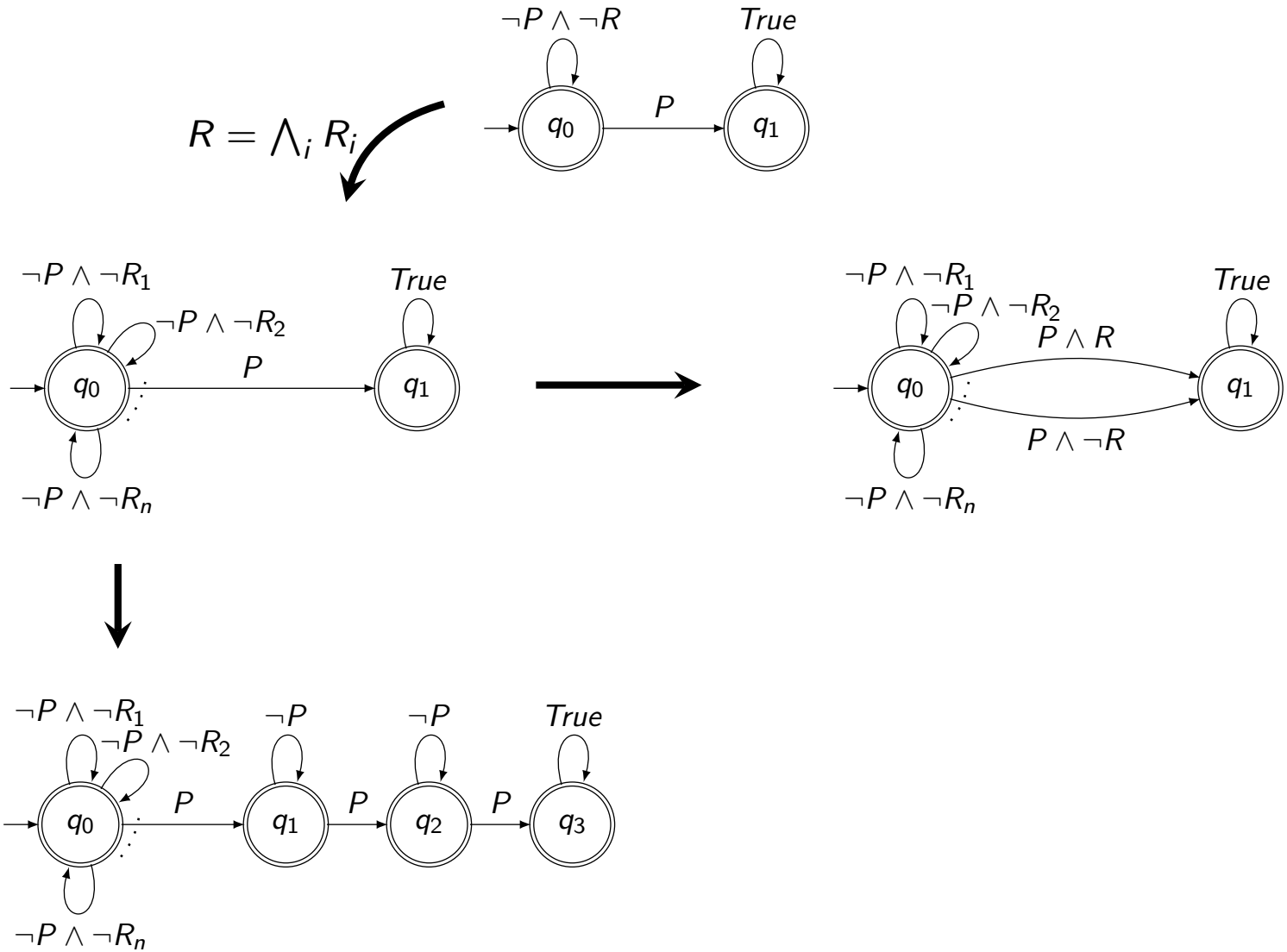

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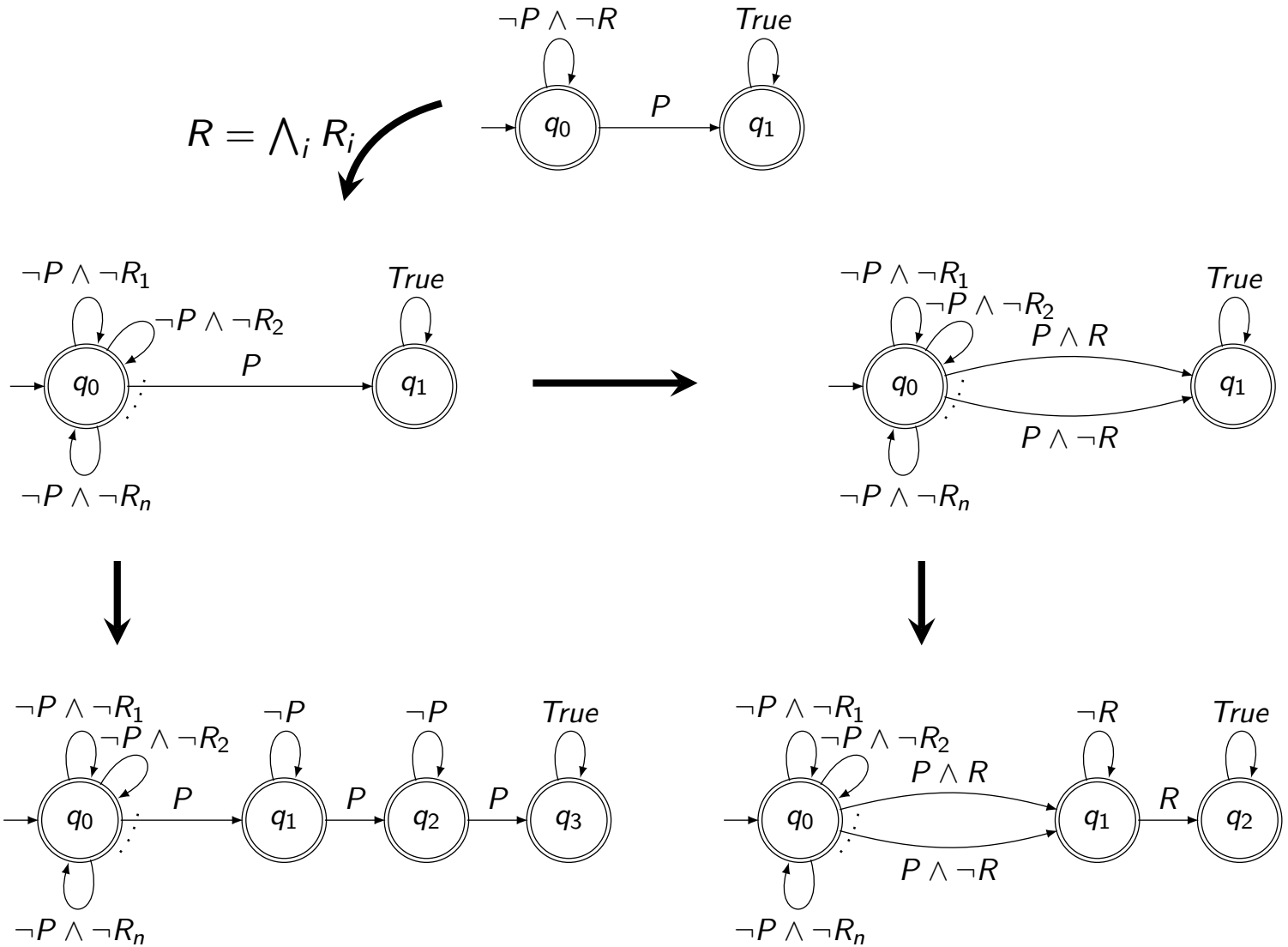
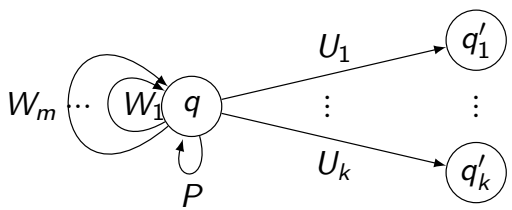


Transformation Rule

Transformation Definition

- test a reflexive transition $\xrightarrow{P}$
- predicate coverage : test all predicate cases $P = (\overbrace{(\bigvee_i R_i)}^{\mathcal{R}} \vee \bar{R})$
- paths coverage : unfold $\mathcal{R}$ between 0 and n times



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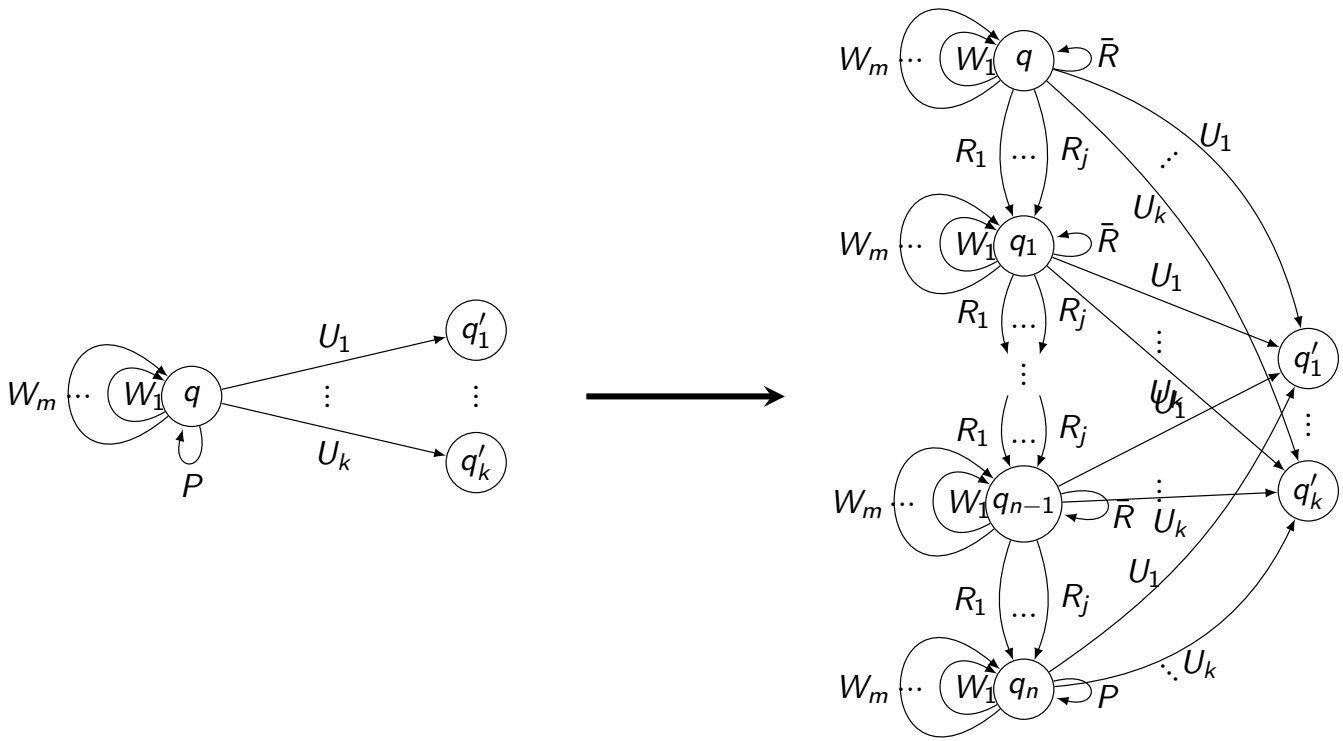

Transformation Realization

1. Define/implement a transformation rule over automata

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Theorem : Composition Consistency

Let $\mathsf{p}_i \stackrel{\text{def}}{=} (Q_{\mathsf{p}_i}, init_{\mathsf{p}_i}, F_{\mathsf{p}_i}, \mathcal{P}, T_{\mathsf{p}_i}),\ i \in \{1,2\}$ be two automata and $\mathsf{s} \stackrel{\text{def}}{=} (Q_{\mathsf{s}}, init_{\mathsf{s}}, F_{\mathsf{s}}, \mathcal{P}, T_{\mathsf{s}})$ be any scope automaton. Then, we have :

$$\left.\begin{array}{rcl} \mathcal{L}(\mathsf{p}_1) & = & \mathcal{L}(\mathsf{p}_2) \\ \mathcal{L}^{\omega}(\mathsf{p}_1) & = & \mathcal{L}^{\omega}(\mathsf{p}_2) \end{array}\right\} \iff \forall \mathsf{s}.\ \mathcal{L}^{\omega}(\mathsf{p}_1 \odot \mathsf{s}) = \mathcal{L}^{\omega}(\mathsf{p}_2 \odot \mathsf{s})$$

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$$\left.\begin{array}{rcl} \mathcal{L}(p_1) & = & \mathcal{L}(p_2) \\ \mathcal{L}^\omega(p_1) & = & \mathcal{L}^\omega(p_2) \end{array}\right\} \iff \forall s.\ \mathcal{L}^\omega(p_1 \odot s) = \mathcal{L}^\omega(p_2 \odot s)$$

3. Prove transformation consistency !

Thank you

any questions ?

References

Dwyer, M., Avrunin, G. S., and Corbett, J. C. (1999).
Patterns in property specifications for finite-state verification.
In Proceedings of the 21st International Conference on Software Engineering, pages 411–420.

Taha, S., Julliand, J., Dadeau, F., Castillos, K. C., and Kanso, B. (2015).
A compositional automata-based semantics and preserving transformation rules for testing property patterns.
Formal Asp. Comput., 27(4) :641–664.